SUPPLEMENTAL INFORMATION: OPTIMIZATION WITH FOCUSSING LENSES

The measurements described in this paper used wide lenses that cover the entire multi-beam. An array of small lenses (each covering a single beam) is another alternative for use on multi-beam VSANS instruments. Lenses that would be smaller with 2.5 mm (instead of 25 mm) curvature and 2.5 mm (instead of 25 mm) diameter, but a smaller number close to 3 (instead of 30) lenses would be stacked along the beam for focusing around 14 Å neutron wavelength. In order to estimate potential gains in using such an array of lenses, consider the “transmission” factor for the beam-defining multi-hole sample aperture which is the ratio $T_2 = \pi r_2^2 / a_x a_y$ of the open area for each hole (πr_2^2) to the total area ($a_x a_y$). This ratio ranges between 0 and $\pi/4$. Given the described parameters (r_2 = 0.45 mm, a_x = 2.24 mm, a_y = 2.6 mm), this ratio is around 11%. This is a rough estimate. Opening up the sample aperture holes to r_2 = 0.9 mm without a change in center-to-center inter-hole distances would raise the transmission ratio to roughly 44%. The configuration of the intermediate apertures would have to be adjusted in order to avoid cross collimation. It is not clear whether this option is viable since the overkill apertures are already tightly packed. Another option is discussed next.

Assume that sample term in the spatial variance (in equation 6 of the main paper) when lenses are used becomes a fractional portion ν (with $\nu \geq 1$) of its value without lenses; i.e., $\frac{2}{3}\left(\frac{\Delta\lambda}{\lambda}\right)^2 r'^2_2 = r_2^2 \frac{1}{\nu}$, so that the radius of an individual hole on the source aperture (with lenses) becomes

$$r'^2_2 = \frac{3}{2}\left(\frac{\lambda}{\Delta\lambda}\right)^2 r_2^2 \frac{1}{\nu} \qquad \text{(SI1)}$$

If we sssume that the overall variance remains unchanged (with and without lenses), then the radius of the individual hole on the source aperture (with lenses) becomes

$$r'^2_1 = r_1^2 + \left[\frac{L_1 + L_2}{L_2}\right]^2 r_2^2 \left(1 - \frac{1}{\nu}\right) \qquad \text{(SI2)}$$

We now assume that the transmission of the source aperture remains unchanged (without and with lenses), that is $T_1' = T_1$, or explicitly

$$\frac{\pi r_1'^2}{a_{1x}' a_{1y}'} = \frac{\pi r_1^2}{a_{1x} a_{1y}} \qquad \text{(SI3)}$$

Knowing that the source aperture total width and height are $W_1 = n_{1x} a_{1x}$, $H_1 = n_{1y} a_{1y}$, and keeping the same aperture sizes (with and without lenses) so that $W_1' = W_1$ and $H_1' = H_1$, then $n_{1x}' = n_{1x}\, r_1/r_1'$ and $n_{1y}' = n_{1y}\, r_1/r_1'$.

Moreover, assuming that the transmission of the sample aperture remains the same or (at least) increases gives another relation $T_2' = T_2 \mu$ (with $\mu \geq 1$), or explicitly

$$\frac{\pi r_2'^2}{a_{2x}' a_{2y}'} = \frac{\pi r_2^2}{a_{2x} a_{2y}} \mu \qquad \text{(SI4)}$$

Along with $W_2' = n_{1x}' a_{2x}'$ and $H_2' = n_{1y}' a_{2y}'$ where we have imposed the same number of holes on the source and sample apertures $n_{1x}' = n_{2x}'$ and $n_{1y}' = n_{2y}'$, we therefore obtain the relations

$$a_{2x}' = a_{2x} \frac{r_2'}{r_2} \frac{1}{\sqrt{\mu}} \text{ and } a_{2y}' = a_{2y} \frac{r_2'}{r_2} \frac{1}{\sqrt{\mu}} \qquad \text{(SI5)}$$

Using these estimates, we can obtain optimized aperture configurations in specific cases as shown in Table 1. The total open source area SoA ($= \pi r_1'^2\, n_{1x}' n_{1y}'$) and sample area SaA ($= \pi r_2'^2\, n_{2x}' n_{2y}'$) are also included. The source-to-sample and sample-to-detector distances used are L_1=3730 mm, L_2=6078 mm, and the wavelength spread is $\Delta\lambda/\lambda$=0.15, as before.

Table 1: Optimized aperture configurations

Cases	Source Aperture (sizes in mm)	Sample Aperture (sizes in mm)
Without lenses	r_1=0.64, n_{1x}=15, n_{1y}=40, a_{1x}’= 1.67, a_{1y}’= 1.87, W_1=25, H_1=75, SoA=772 mm^2	r_2=0.45, n_{2x}=15, n_{2y}=40, a_{2x}’= 0.83, a_{2y}’= 0.94, W_1=12.5, H_2=37.5, SaA=382 mm^2
With lenses ν=1, μ=1	r_1’=0.64, n_{1x}’=15, n_{1y}’=40, a_{1x}’=1.67, a_{1y}’=1.87, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=3.67, n_{2x}’=15, n_{2y}’=40, a_{2x}’=6.8, a_{2y}’=7.65, W_2’=102, H_2’=306, SaA==25447 mm^2
With lenses ν=2, μ=1	r_1’=0.82, n_{1x}’=12, n_{1y}’=31, a_{1x}’=2.14, a_{1y}’=2.40, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=2.60, n_{2x}’=12, n_{2y}’=31, a_{2x}’=4.81, a_{2y}’=5.41, W_2’=56.29, H_2’=168.87, SaA=7741 mm^2
With lenses ν=3, μ=1	r_1’=0.87, n_{1x}’=11, n_{1y}’=29, a_{1x}’=2.27, a_{1y}’=2.56, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=2.12, n_{2x}’=11, n_{2y}’=29, a_{2x}’=3.93, a_{2y}’=4.41, W_2’=43.23, H_2’=129.68, SaA=4565 mm^2
With lenses ν=4, μ=1	r_1’=0.90, n_{1x}’=11, n_{1y}’=29, a_{1x}’=2.34, a_{1y}’=2.63, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=1.84, n_{2x}’=11, n_{2y}’=29, a_{2x}’=3.40, a_{2y}’=3.83, W_2’=36.40, H_2’=109.20, SaA=3237 mm^2
With lenses ν=4, μ=2	r_1’=0.90, n_{1x}’=11, n_{1y}’=29, a_{1x}’=2.34, a_{1y}’=2.63, W_1’=25, H_1’=75, SoA=772 mm^2	r2’=1.84, n2x’=11, n2y’=29, a2x’=2.40, a2y’=2.71, W_2’=25.74, H_2’=77.21, SaA=3237 mm^2
With lenses ν=4, μ=3	r1’=0.90, n1x’=11, n1y’=29, a1x’=2.34, a1y’=2.63, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=1.84, n_{2x}’=11, n_{2y}’=29, a_{2x}’=1.96, a_{2y}’=2.21, W_2’=21.02, H_2’=63.05, SaA=3237 mm^2
With lenses ν=4, μ=4	r_1’=0.90, n_{1x}’=11, n_{1y}’=29, a_{1x}’=2.34, a_{1y}’=2.63, W_1’=25, H_1’=75, SoA=772 mm^2	r_2’=1.84, n_{2x}’=11, n_{2y}’=29, a_{2x}’=1.70, a_{2y}’=1.91, W_2’=18.2, H_2’=54.60, SaA=3237 mm^2

The four cases corresponding to μ=1 with different values of ν involve the same overkill aperture configuration with and without lenses. The gain stems from the fact that the sample dimensions are larger. The gain can be estimated as the ratio of SaA with lenses over SaA without lenses. The case for $\nu = 4$ and $\mu = 1$, for example, corresponds to a gain of 8.47. The last three cases (with increasing μ for a given value of ν) correspond to slightly smaller sample sizes but require the addition of overkill apertures.

Note that moderate losses are expected due to finite neutron transmission of the lens system. For example, the transmission of the 30 lens system described in the early part of this paper is around 73 % for 6 Å wavelength neutrons. The transmission of the proposed lens system consisting of an array of lenses with 3 smaller lenses stacked for each hole would be around 98 % for the same wavelength.

In conclusion, when no lenses are used, the converging collimation imposes that the size of the sample aperture be close to half that of the source aperture, while when lenses are used, the sample aperture can be as large as (or larger than) the source aperture. Increase in sample size is accompanied by neutron gain.