Louis Nirenberg (1925–2020)

Coordinated by Robert V. Kohn and Yanyan Li

Figure 1. Louis Nirenberg, circa 1991.

Introduction

Louis Nirenberg died in Manhattan on January 26, 2020, at the age of 94. He was among the most influential analysts of the twentieth century. A leading expert in partial differential equations, his fundamental contributions had major impacts in many areas including geometry, complex analysis, and fluid dynamics. Nirenberg's influence reflected his success in solving important problems, identifying fruitful directions, and introducing new techniques.

Robert V. Kohn is a Silver Professor of Mathematics at the Courant Institute of Mathematical Sciences, New York University. His email address is kohn@cims.nyu.edu.

Yanyan Li is a Distinguished Professor of Mathematics at Rutgers University. His email address is yyli@math.rutgers.edu.

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He was also a great friend and role model to a worldwide community of students, colleagues, and collaborators.

The diversity of his contributions makes them difficult to summarize. Without attempting completeness, some favorites are: the solution in his PhD thesis of the Weyl and Minkowski problems, which rested on an a priori estimate for fully nonlinear elliptic PDE in two space dimensions; the Gagliardo-Nirenberg inequality in the 50s; the Newlander-Nirenberg theorem in the 50s, on the integrability of almost complex structures; his work in the 50s and 60s with Agmon and Douglis, which achieved a comprehensive understanding of regularity up to the boundary for linear elliptic equations and systems; his work in the 60s with F. John, which introduced the space of functions with bounded mean oscillation (BMO) and established the John-Nirenberg inequality for BMO functions; the concept of a pseudodifferential operator, which he introduced with J. J. Kohn in the mid-60s; his work with C. Loewner in the 70s, which solved a problem from geometry by considering nonlinear PDEs invariant under conformal or projective transformations; the “method of moving planes” (introduced in the 70s with B. Gidas and W.-M. Ni) and the “sliding method” (introduced in the 80s with H. Berestycki), which address the symmetry of positive solutions to nonlinear PDEs using subtle applications of the maximum principle; his work in the 80s with H. Brezis, concerning the existence and nonexistence of solutions to semilinear elliptic equations with critical exponents; and his work with L. Caffarelli and J. Spruck in the 80s concerning the Monge-Ampère equation and other fully nonlinear elliptic equations.

The focus of this Memorial Tribute is, however, not on Louis' mathematical

Figure 2. Louis Nirenberg, 1957.

Figure 3. John F. Nash and Louis Nirenberg receiving the 2015 Abel Prize from His Majesty King Harald at the award ceremony. Oslo, May 19, 2015.

contributions. They have been reviewed elsewhere, including [2–4, 9, 10, 12], and additional articles on his work are in preparation. Rather, the focus here is on Louis as a mentor, collaborator, colleague, and friend.

His mathematical studies began at McGill University, from which Louis graduated in 1945 with a degree in Mathematics and Physics. Peter Lax's contribution to this article tells the story of how Louis came to do a Masters degree and then a PhD at New York University. The postwar years were a remarkable time for mathematics at NYU—his fellow students included Avron Douglis, Harold Grad, Eugene Isaacson, Joseph Keller, Martin Kruskal, Peter Lax, and Cathleen Morawetz. Louis remained at NYU for his entire career: after completing his PhD in 1949 with guidance from James Stoker and Kurt Friedrichs, he held a two-year postdoctoral position then joined the faculty in 1951. His title was Professor of Mathematics from 1957 until 1999, when he retired and became Professor Emeritus.

In recognition of his contributions to mathematics Louis received many prestigious awards, including the AMS's Bôcher Memorial Prize (1959), the Crafoord Prize (1982), the Canadian Mathematical Society's Jeffery-Williams Prize (1987), the AMS's Leroy P. Steele Prize for Lifetime Achievement (1994), the National Medal of Science (1995), the International Mathematical Union's Chern Medal (2010), the AMS's Leroy P. Steele Prize for Seminal Contribution to Research (2014), and the Abel Prize (2015). He became a member of the American Academy of Arts and Sciences in 1965, and a member of the National Academy of Sciences in 1969.

Additional information on Louis' career can be found in his cv [5] and his autobiography [11]. Readers may also be interested in the oral history available at the Simons Foundation's Science Lives website [1] and an extensive interview published in the *Notices* of the AMS in 2002 [6].

Figure 4. David Dinkins (then Mayor of New York) with Louis and his wife Susan Nirenberg in the early 90s, when Louis received the New York City Mayor's Award for Excellence in Science and Technology.

Peter Lax

Louis Nirenberg's parents came from the Ukraine and settled in Canada where Louis was born and grew up. Louis got his bachelors degree at McGill University in Montreal where one of his friends and colleagues was Sara Courant, who was engaged to be married to Ernest, Richard Courant's eldest son. Sara told her father-in-law about the brilliant young scientist studying in Montreal. Courant, who was always on the lookout for talented young scientists, invited Louis to New York City. At that time Louis planned a career in physics. Courant told Louis that a Masters degree in mathematics would be a good preparation for a physicist. Louis took Richard's advice and once Louis had embarked on the study of mathematics his physics career faded.

Jim Stoker was Louis' thesis advisor and he suggested an important unsolved problem as a doctoral dissertation, which is very unusual. Louis solved it, which is even more unusual. (Kurt Friedrichs made some helpful suggestions.)

Louis and I met in 1946 and soon thereafter we became best friends. My wife Anneli and I adored both Louis and his wife Susan. We traveled together, and spent our favorite holiday, Thanksgiving, together with our families and with foreign visitors at the Institute, introducing them to the unique American celebration which became our favorite. Louis was an extraordinary host and Sue was a world-class cook. Our children were close to both Louis and Susan,

Peter Lax is an emeritus professor of mathematics at the Courant Institute of Mathematical Sciences, New York University. His email address is lax@cims.nyu.edu.

Figure 5. Louis with Peter Lax, 1985.

whom they referred to as their aunt and uncle and we were all close with Marc and Lisa, their children.

Louis' interests extended way beyond just mathematics. He enjoyed worldwide traveling, even into his 90s in a wheelchair.

He was a film aficionado. When we were young, and they existed, he much preferred going to double features. When I would ask him if he was interested in coming to a film with me, he'd reply "What's with it?" More recently he recuperated from a medical procedure at a facility on West Houston Street. I heard my son, Jim, tell him he had chosen it because of its proximity to Film Forum. I thought Jim was joking but later learned that Louis would go to matinees when the physical therapy got slow. Probably it was the best PT he could have gotten.

Cuisine was another great interest of Louis. A couple of months before he passed away, Louis took us to a great Turkish restaurant which he'd recently discovered on the upper west side. One could ask him to recommend any cuisine in any city and expect a list of suggestions and recommended dishes and when best to dine at a given restaurant.

Figure 6. Louis with Nanette Aubin. Rome, 2008.

Louis was extremely witty and a terrific raconteur; a grand master of telling Jewish jokes. He repeated an observation made by a linguist friend who remarked "It's very strange, but when you tell jokes you often have a slight Hungarian accent." I was extremely proud and flattered.

Louis loved art, and mentioned just a few months ago that a visit to the Met had restored his mood.

Music, from classical to popular to esoteric world compositions, intrigued him. Louis told me that he tried to appreciate modern Japanese music. He bought a record and

played it for Susan, asking for her reaction. Sue replied "It might be good for speeding departing guests."

Louis truly loved people, and had more friends than anyone I know. During a recent visit, the phone was ringing with friends calling from around the world, and others dropping by. I can't mention all of his wonderful friends, but Jalal Shatah is one who epitomized the best Courant collegiality and who was a close, helpful, wonderful friend and supporter of Louis over the years.

Of course it was his 20-year relationship with Nanette (Jeanne Aubin) which was most important to him and gave him the strength to surmount many physical and medical challenges over the last several years. A few years ago Louis told me that he attended Nanette's birthday celebration in France, hosted by her large and loving family. Her son acknowledged Louis, calling him a "lucky" man to have found Nanette. Louis replied, "Yes, and Nanette is lucky to have found me, because I am a lucky man."

Shmuel Agmon

I met Louis for the first time in the summer of 1952, in the old Graduate School of NYU. It was a casual meeting. Louis was telling me enthusiastically about his visit to Europe, from which he had just returned. After a while, we entered an empty room to discuss mathematics. This resulted in a joint paper, together with Protter, the first in a series of collaborations that Louis was about to have with many friends and colleagues.

Louis had finished his PhD three years earlier. I was on my way back to Israel after three years at Rice University. My knowledge of partial differential equations was scant. I came to the subject quite late, and was self-taught. Fortunately, in the summer of 1951, at UCLA, I met Lipman Bers, who got me interested in the Tricomi problem. He invited me to spend a few months at NYU on my way to Israel. This is how I first made contact with Louis.

We met again in the fall of 1954 in Trieste (which was still under international jurisdiction), where Courant and Fichera had organized one of the first conferences in PDEs after the war. But it was mainly in 1955–1956 when our friendship flourished. I was on leave from the Hebrew University, at NYU. It was a special year. The math department had just moved to a new location, and there were many visitors: Lars Hörmander, at age 25, visited the USA for his first time, John Nash often joined from Princeton, and several others. Louis and I, together with Avron Douglis, discussed a joint project, namely: developing a theory of elliptic boundary value problems of arbitrary order. The

Shmuel Agmon is an emeritus professor of mathematics at Hebrew University of Jerusalem. His email address is agmon@math.huji.ac.il.

Figure 7. Left to right: Shmuel Agmon, Hans Weinberger, Louis Nirenberg. Minneapolis, around 1980.

time was ripe for that, and we had some vague ideas how to attack the problem. However, the actual collaboration started only towards the end of that year, and most of the work was done in 1957–1958 by correspondence. This was by far our largest collaboration, and is much cited.

Louis spent 1958–1959 in Italy, where we met again at a conference in Pisa, attended by Bers, Stampacchia, and Magenes, among others. Each of the four days of this memorable meeting was held at a different location. Louis was very fond of Italy, and the Italians reciprocated, and liked him very much. He was a fantastic speaker, capable of explaining complicated things in a simple way. The same impact was felt years later, when he participated in a historic joint US-Soviet conference on PDEs in Novosibirsk, in 1963.

In 1960 Louis visited Israel for a Symposium on Linear Spaces, the first international conference to be held in the young state of Israel. His last visit to Israel was in 2016, to attend a tribute at the Technion, celebrating his receipt of the Abel Prize. Despite his age, he was still full of joy of life.

In 1960–1961 I spent my second sabbatical at the Courant Institute, and did some more work with Louis. In later years we maintained close ties, but our work diverged. Louis turned more to hard problems involving nonlinear PDEs such as the Navier-Stokes and Monge-Ampère equations. I, in turn, became interested in problems involving the Schrödinger equation and scattering theory.

I had many friends, but Louis (and Lipa Bers) were my closest. Louis was a warm person, who loved his city

Figure 8. A letter from Louis Nirenberg to Djairo Guedes de Figueiredo, with suggestions and encouragement. Early 1960s.

New York, and enjoyed showing her wonders to the overwhelmed visitor. He and his wife Sue, the daughter of a Hebrew writer, were deeply embedded in the cultural life of New York. When I was staying in their apartment, I always looked forward to reading the latest issue of the *Saturday Review*.

Louis mentions in his biography [11] his three “Fratelli”: Guido Stampacchia, Peter Lax, and myself. This distinguished title was a great honor. I have deeply cherished his friendship.

Djairo Guedes de Figueiredo

The hardest part in reminiscing about Louis Nirenberg is to select amongst so many influential interactions.

As I fondly look back at my graduate years (1957–61) at New York University, I vividly recall the excitement in the environment at the Institute of Mathematical Sciences, shortly afterwards named after Richard Courant, and located, at the time, in a charming building on Waverly Place. As a foreign student, arriving from Brazil where mathematics was in an embryonic stage, I took all of it in and it indelibly marked my career.

My initial goal for my doctoral work was in probability theory. However, as I like to say metaphorically, Nirenberg together with his colleagues K. O. Friedrichs, Fritz John, Peter Lax, and Lipman Bers laid a magical spell on students, attracting them to the area of partial differential equations. They were, if I may say so, the Wizards of Waverly Place.

Djairo Guedes de Figueiredo is an emeritus professor of mathematics at the University of Campinas. His email address is djairo@unicamp.br.

Nirenberg's enthusiasm and ability to unveil the beauty of PDEs is, to this day, branded in my mind. You see, this is what great advisors do. They instill in their advisees this passion for the subject and win over not only their minds but their hearts.

Nirenberg was certainly an influential advisor: he pressed his students to excel in their work and to develop independently. However, he combined this characteristic with accompanying them through frequent meetings and lively interest. Whenever he travelled, which he regularly did, he would ask someone else to meet with his students. I recall Martin Schechter following up on my work in such occasions.

After the conclusion of my doctoral work, inspired by those years at NYU, I returned to Brazil full of ideals and determined to consolidate the field of partial differential equations in my homeland. I was hired in 1962 by the University of Brasilia, located in the newly founded capital, where I was entrusted with the project of establishing the Mathematics Department. In this setting, pursuing mathematical research was quite challenging, to say the least, not only because mathematics, as a field, was just starting to take root in Brazil, but also on account of the huge load of administrative duties that ensued.

At this stage, Louis was a great friend and mentor. Aware of this situation, he took great interest in my mathematical development and expressed it in a series of letters he wrote to me at that time, with suggestions of problems to investigate that were related to my prior work. This was all the more significant since he once mentioned that he did not like writing letters. He also encouraged me to strive in my teaching and advising, inspiring me to establish the first Mathematics Master's Degree Program in Brazil. I can say that this correspondence was always full of encouragement, which was much needed at the time.

In subsequent years, political unrest emerged in Brazil and shortly thereafter, in 1964, a military coup took place. Academically, the situation at the University of Brasilia was precarious and I applied for and was awarded a Guggenheim Fellowship to spend a year in the US. At the conclusion of that fellowship, once again, Louis took a great interest in helping me to find a position in the US, since returning to Brazil would have stifled my attempts to pursue research. This was the kind of person he was! (Eventually the situation in Brazil improved, and I returned to the University of Brasilia in 1971.)

Throughout the years, I met Louis on numerous occasions, including at meetings in Europe, where we enjoyed many pleasant discussions. Now, as I look back, I know I was privileged to have had him as my advisor and friend.

Figure 9. S. R. Srinivasa Varadhan, Louis Nirenberg, Vasundara Varadhan. Jardin du Luxembourg, 2017.

S. R. Srinivasa Varadhan

I came to Courant as a visiting member in the fall of 1963. Soon after that, at a seminar, someone pointed out a distinguished looking man with a beard and said "That is Nirenberg." I was a probabilist, still unfamiliar with the world of PDEs. While the name was unfamiliar to me, I was sure anyone looking that distinguished must indeed be a great mathematician. The Institute was divided between 4 Washington Place and 25 Waverly Place and I only saw Louis at some seminars and had very little contact with him. But it all changed when Warren Weaver Hall opened in the spring of 1965.

In the fall of that year, Louis and I met in his office to discuss some mathematics. I had proved what was really a PDE result, but by using probabilistic arguments. Louis told me how I could simplify the proof by using the maximum principle. It opened my eyes and taught me how to exploit this connection both ways. Dan Stroock and I worked together for some years on problems that straddled PDEs and probability. Eventually Louis, Henri Berestycki, and I worked together on a project and have a joint publication. The lounge at Warren Weaver Hall was the place where faculty, visitors, postdocs, and graduate students met informally for lunch, tea, and conversation. Louis, Peter, Jürgen, Harold, Fritz, and Mony were regulars.

Louis was a movie buff and he loved music. He introduced us to the World Music Institute, which showcased performers from around the world. Louis and Sue were excellent hosts and would often invite younger colleagues for dinner parties at their home on the upper west side. Vasu and I have spent many evenings at the Nirenbergs' enjoying Sue's excellent cooking. Louis loved going out to restaurants and trying different cuisines and we would

S. R. Srinivasa Varadhan is the Frank Jay Gould Professor of Science at the Courant Institute of Mathematical Sciences, New York University. His email address is varadhan@cims.nyu.edu.

often join him. Sue passed away and after a few years Louis and Nanette became companions, splitting their time between Paris and New York. Louis had retired by this time. The tradition of having dinner parties and going out to restaurants continued, however, mostly in New York but occasionally in Paris as well.

Louis' health deteriorated and he was confined to a wheelchair. Although he was in and out of hospitals and nursing homes, it did not dampen his enthusiasm for life. He continued to come to the Institute for seminars, entertain at home, and join friends for dinners at their homes or in restaurants. Vasu and I will miss him dearly.

Joseph J. Kohn

I have known Louis Nirenberg since I was a graduate student at Princeton in the 1950s. My thesis advisor, Don Spencer, frequently consulted Louis in connection with his joint work with Kodaira on the deformation of complex structures. I was privileged to attend many of their meetings. Louis' participation was truly remarkable. Very often he would wade into some seemingly intractable problem, find its critical PDE component and proceed masterfully to prove the estimates which were key to it. His skill and clarity were much admired. Spencer once asked Louis: "How do you do it?" Louis replied, "It's just calculus." Spencer sighed and said "Louis, I wish I had a copy of your calculus book."

In the early 1960s Louis became interested in my work on the $\bar{\partial}$ -Neumann problem and we embarked on a project to apply and expand it. Collaborating with Louis was exciting, challenging, and also very pleasant. After several hours of work during a weekend Louis would often suggest that we go to a movie. (Louis loved movies; he was a bona fide film critic; he had seen an enormous number and remembered each plot in all

Figure 10. The domain described in the text; its equation is $Re(w) + |z|^8 + \frac{15}{7}|z|^2 Re(z^6) = 0$.

its details, the cinematography, and the names of all the actors and directors.) If we didn't find a movie that looked promising Louis would suggest that we go to a double feature, as watching two mediocre films for the price of one would surely make the endeavor worthwhile. Apart from the work, the movies, and the great conversations, there

were many outstanding meals. Louis had a knack for discovering Manhattan's hidden hole-in-the-wall gems.

A great part of our work was concerned with the study of systems of subelliptic equations. This work led us and others to discoveries in various directions: several complex variables, algebraic geometry, microlocal analysis, harmonic analysis, etc. For example, our work on the regularity of these equations led us to develop a theory of pseudodifferential operators. We also discovered an interesting domain in two complex variables, which was illustrated on an MSRI T-shirt (see Figure 10). We came up with this domain while searching for explicit representations of solutions to the $\bar{\partial}$ -Neumann problem on a domain in several complex variables. Such representations usually involve local holomorphic functions, defined in a neighborhood of a boundary point which vanish at that point but not in the domain. We were surprised to find a domain on which the $\bar{\partial}$ -Neumann problem has a solution, but the boundary of that domain contained a point such that if any local holomorphic function vanishes on it, then it must also vanish at some points both inside and outside of the domain [8].

Louis had an irrepressible sense of humor. His knowledge of jokes (especially Jewish ones) was encyclopedic. He frequently found the humorous aspect in the most unlikely situations. We once wrote a paper that included a rather long and cumbersome calculation; following it Louis inserted on p. 844: "With these tedious estimates, as well as some readers, behind us, we may proceed to deal with ..." [7].

Louis had very broad cultural interests and these were combined with a passion for travel. I was fortunate to observe this firsthand on many occasions. Before a trip he took to China in the 1970s, he drew up a list of things that he wanted to see. In Beijing he asked his guide to take him to one of them, a renowned building. However, this building was not on the guide's list of permitted places and so he told Louis that he had no idea how to get there. Louis knew the exact location and instructed the guide to take him there.

It was my great fortune to be Louis' friend. I admired him greatly, not only for his stellar research and his remarkable talent for teaching and explaining so much so clearly and at so many different levels, but also for his great humanity.

Joseph J. Kohn is an emeritus professor of mathematics at Princeton University. His email address is kohn@princeton.edu.

Haim Brezis

I first met Louis in Paris at a conference during the spring of 1967. Shortly thereafter he invited me to visit the Courant Institute for the academic year 1968–69. Since my spoken English was limited and tainted with a heavy French accent, one of my first tasks upon arrival in New York was to register for an English conversation class at NYU. I told Louis proudly that I was working on my English. To my surprise he seemed to be disappointed and even discouraged me! Louis pointed out that it would be much easier for me, a 24-year-old bachelor, to date American girls if I kept my French accent ... I was baffled! No one in the French academic circles had ever interfered with my romantic encounters. This was typical Louis! Over the years he would shape my life much beyond mathematics.

Louis had a great love and admiration for Italy, which he was always eager to share. In fact, one of our earliest meaningful meetings took place in Rome in the spring of 1968. At that time, I was visiting Stampacchia in Pisa, and he suggested that I accompany him to Rome where an international conference was taking place. I gladly accepted since I had never been to Rome. Guido and I reached Rome late in the evening and immediately met Louis who had just arrived from New York. I was privileged to witness an event which has unfortunately disappeared in our age of emails. Two great mathematicians were excitedly sharing the latest discoveries from Pisa and New York. After about an hour Guido turned to Louis and said, pointing to me: “This young man has never been in Rome before. Where shall we start our tour?” Louis replied without hesitation: “Piazza del Campidoglio.” It was past midnight when the three of us reached this majestic square and admired the breathtaking view overlooking the Foro Romano. On the next day, after a long series of lectures, Louis decided that I should join him to see a painting exhibit at the Villa Borghese, followed by a dinner at a restaurant in the ghetto where Guido and Louis introduced me to the famous “Carciofi alla Giudia.” By the end of the conference I was ready to write a guide book about Rome. This was just the beginning. Over the years, Louis provided me with long lists of places I must see or stay in, from London to Montreal, from Bali to Damascus.

Working with Louis was a unique experience. When we were stuck on a problem Louis would propose a break, for coffee, or a stroll in the Village, or a stop at the famous Strand bookstore. Returning to his office, Louis was again

Haim Brezis is an emeritus professor of mathematics at Université Paris-Sorbonne, and a Visiting Distinguished Professor at Rutgers University and Technion-Israel Institute of Technology. His email address is brezis@math.rutgers.edu.

Figure 11. Haim Brezis and Louis Nirenberg in a mathematical discussion. Jerusalem, January 2002.

full of energy and ideas. When I was in Paris and wanted to urgently discuss a thought with Louis in New York, I learned to control myself and never call before 5 p.m. (Paris time). Louis often saw movies in the evenings and then he could work late into the night. Regularly he would share his impressions about movies and was disappointed with my limited cinematographic knowledge.

Louis often complained—even when he was young—that his memory was fading. I postulate that he was gifted with an extraordinary “compensation mechanism.” One day I proposed at some point in our discussion, to apply the celebrated Sobolev inequality, and I wrote down on the blackboard how it would impact our problem. Louis moved to the other end of his huge blackboard and started scribbling some formulas. After he emerged, Louis confessed that he never remembered the inequality and had to reconstruct it each time he needed it. It turns out that in 1959, Louis had discovered the most elegant proof for the Sobolev inequality; it appears in numerous textbooks and is often associated with Louis’ name. Can you imagine Einstein unable to remember $E = mc^2$? This may explain why Louis kept a youthful enthusiasm for mathematics throughout his entire life. He was never blasé and always eager to gaze with fresh eyes at old problems.

Paul H. Rabinowitz

Louis was one of my heroes. I first met him formally when I was working on my thesis at NYU under the direction of Jürgen Moser. Louis made some suggestions that were very helpful to me and later was one of the official readers of the thesis. Subsequently, in the mid-1970s when Louis

Paul H. Rabinowitz is an emeritus professor of mathematics at the University of Wisconsin-Madison. His email address is rabinowitz@math.wisc.edu.

shifted his research interests to nonlinear PDEs, I met him frequently at conferences and got to know him well. Louis was wonderful company. A colorful raconteur and avid joke teller, Louis had an appropriate story for every occasion. In addition to mathematics, he was a source of useful information. If you wanted a recommendation for a good restaurant in New York, Rome, Paris, anywhere, just ask Louis. If you needed a suggestion for a museum to visit, a book, a music CD, or especially a movie, just ask Louis. He was a movie buff. Indeed as the story goes, one day he came home from the Courant Institute looking a bit tired and his wife, Sue, asked "Rough day?" He replied "Yes, there was a double feature at the cinema today."

In 1982, Louis was awarded the first Crafoord Prize in Mathematics jointly with Vladimir Arnold. I had the good fortune of being invited to the symposium attached to the prize ceremony. The prize was presented by the King of Sweden. Some of us envisioned a headline in the *Svenska Dagbladet* reading "King of Sweden meets King of Calculus Inequalities." As I recall it, at the conference or the associated banquet, Louis was asked to say a few words. Having learned that research budgets were about to be cut in Sweden, he included an impassioned plea on the importance of supporting research.

When Louis was invited to give a lecture, occasionally he opted to speak on the work of someone else that intrigued him. This gives me an opportunity to illustrate Louis' quick wit and story telling. A caveat: I remember the background a bit differently than Louis did so you will get my version, but the story is the same. Some time in the late 1970s, as part of a longer trip, Louis came to Madison to give a seminar talk. I asked him about the rest of his trip and he told me he would give a colloquium in Berkeley in a couple of days. I replied "That's a coincidence. I'm doing the same next week. What are you going to speak on?" Louis said he planned to give a talk on a recent paper of mine. Since I had intended to do the same, we had a dilemma, but discussed matters and arrived at a satisfactory *modus operandi*. Then Louis told me the following story:

There is a writers club in Moscow where showing a foreign film is a weekly event. The film is the original version so it is neither dubbed nor with subtitles. A member of the club gives a simultaneous translation into English as the film is shown. One particular week, there was a French film. The regular French translator was not available so a volunteer was sought from the audience. Someone obliged. In fact he didn't know much French, but simply made up a story in a very charming and clever fashion. At one point, a man seemed to be marrying his sister but somehow the translator got around this. The audience loved the movie and insisted that it be shown again the

following week. This time the regular translator was present. It was a disaster; the audience booed him.

Louis then suggested that I begin my talk with this story. And I did!

Louis, you will be missed!

Charles Fefferman

Like every other analyst, I greatly admired and was strongly influenced by Louis. That influence is especially obvious in the direct line from the John-Nirenberg inequality through Eli Stein's result on the boundedness of singular integrals from L^∞ to BMO, to my work on the duality of H^1 and BMO.

Even more obvious is the influence on me of Louis' work with François Trèves on local solvability of linear PDEs of principal type. Richard Beals and I proved that the Nirenberg-Trèves condition is sufficient for local solvability; Louis and François came up with the relevant conjecture and proved it in many significant cases.

Louis' indirect influence is less obvious but equally strong. In some of Louis' finest work, deep results on PDEs require basic progress in understanding functions and their integrals and derivatives; think of the Gagliardo-Nirenberg inequality. I've tried to do the same thing, usually without success.

Finally, let me mention the Caffarelli-Kohn-Nirenberg theorem on partial regularity for Navier-Stokes, which played a role in sparking my interest in fluid mechanics.

So, like so many others, I feel honored to be asked to pay tribute to Louis Nirenberg. His loss will be keenly felt for a long time.

Shing Tung Yau

I have known Louis Nirenberg since I was a graduate student at Berkeley in 1969. I was in Charles Morrey's class on partial differential equations. I was fascinated both by the story of the Sobolev inequality as well as Louis' elegant proof of the inequality in Morrey's famous book on the calculus of variations. It was not until the summer of 1973 that I met Louis for the first time at a differential geometry conference in Stanford. My teacher S. S. Chern was a good friend of his. In fact, they were finishing a paper on the intrinsic norms of complex manifolds and they needed an estimate for plurisubharmonic functions. At that time, I had just finished my work on some gradient estimates for

Charles Fefferman is the Herbert E. Jones, Jr. '43 University Professor of Mathematics at Princeton University. His email address is cf@math.princeton.edu.

Shing Tung Yau is the William Caspar Graustein Professor of Mathematics at Harvard University. His email address is yau@math.harvard.edu.

harmonic functions on a Riemannian manifold with Ricci curvature bounded from below. This was the first estimate I found and I was very excited about it. But I was not sure whether it was already known to experts or not. Professor Morrey was not doing well because of health problems. Naturally, Louis Nirenberg was the person I wanted to talk to. Chern introduced me to Louis, who, to my surprise, was very nice and humble. He told me that he did not know the estimate. And he started to ask me many questions. His attitude towards a young man reminded me of my first meeting with Chern. During the conference, he attended all of the lectures and took notes seriously, an act that impressed me. Instantly I knew that I had a new teacher on the subject of analysis.

In fact, throughout most of my career, whenever I made progress on nonlinear partial differential equations, I would ask Louis' opinion, and only after he agreed with my calculations did I feel comfortable. In 1974, he and Calabi announced the solvability of the boundary value problem of the real Monge-Ampère equation at the International Congress of Mathematicians in Vancouver. Unfortunately a gap was soon found in the proof. Chern told me the news and suggested that S.-Y. Cheng and I look into it. We had been working on some real Monge-Ampère equations in affine geometry for a while, hence were confident to solve this problem (later we learned that our approach was very similar to theirs). However, after telling him our treatment, he smiled and pointed out a shortfall in our proof. After some hard work, we eventually succeeded in proving existence of a solution which is smooth in the interior of the domain and $C^{1,1}$ up to the boundary. This result was good enough for our application in geometry. Seven or eight years later, with Luis Caffarelli and Joel Spruck, Louis made significant progress on the solvability of the boundary value problem for fully nonlinear elliptic equations. We had a lot of fun talking with him in the process. The ideas I picked up from the study of the real Monge-Ampère equation became highly valuable when I tackled the complex Monge-Ampère equation arising from the Calabi conjecture.

As a matter of fact, when I finally solved the Calabi conjecture in 1976, to make sure of its correctness, I flew from California to see Calabi and Louis. I fondly remember we spent all of Christmas day together going through details of the proof in Louis' office, and having lunch in Chinatown as restaurants elsewhere were closed for the holidays.

We were good friends since then. He was full of humor and cracked jokes all the time. He was more than willing to help budding mathematicians. I always sought his advice in my career and was deeply grateful for his mentorship.

Figure 12. Left to right: Louis Nirenberg, Wei-Ming Ni, David Kinderlehrer. Gainesville, FL, 2005.

I was amazed by Louis' energy and excitement for life and mathematics, which did not diminish when he got older and eventually had to travel in a wheelchair. The last contact I had with him was his request for tickets to attend the graduation ceremony of his granddaughter at Harvard. That was easy. In fact, Harvard respected Louis' contribution to our math department and offered VIP seats for him, but he preferred to stay together with his family. It was a rainy day. I was happy to hold an umbrella for him. After the ceremony, I invited him and I. M. Singer to a nice Chinese dinner. Singer and Louis were good friends. I was really glad to be with my two great mentors. We took some photos together. Somehow I had the bad feeling that this might be the last time we three would meet.

In December 2019, I was told that Louis was critically ill. H. T. Yau and I took a special trip to New York to see him. He was in good spirits. It seemed that his cancer did not bother him much.

After three weeks I received email from him saying that his cancer was completely gone. Unfortunately, he was not able to live much longer. Soon I heard the sad news from Sylvain Cappell that he passed away on January 26, 2020. I am sure that he left this world peacefully. His contribution to mathematics and the math community will last forever.

David Kinderlehrer

Louis, first of all we recognize, was an exquisite mentor. I would stand at the board to explain some idea or something I had done, to have him revisit it with depth and understanding. And the manner of it, so gentle and natural that the "collaboration" was actually a learning experience in many ways. Our initial collaboration was like that. After my awkward attempt to approach a free boundary problem, Louis stared at it through his thick glasses,

David Kinderlehrer is Alumni Professor of Mathematical Sciences and also a professor in the Department of Materials Science and Engineering at Carnegie Mellon University. His email address is daviddk@andrew.cmu.edu.

repurposed it, and our partial hodograph method took flight. He then asked if I knew of Hans Lewy's result on harmonic functions and the reduced wave equation. Taken aback, it hadn't occurred to me that anyone else had read this paper—well, more a little later.

There are many many memories. I recall especially an afternoon lunch in the Lewys' garden, in Berkeley in the early 1980s, with Mrs. Lewy's terrific Italian cuisine and Hans and Louis exchanging (apocryphal) stories and a lot of Chianti. Louis carried a small agenda to record the wine he liked. We had a tour of Mrs. Lewy's atelier. Lewy was the nodal point of our relationship. Louis and Hans were very fond of each other. Some of Louis' work was inspired by Hans, for example, on the Weyl and Minkowski problems and his joint work with François Trèves on solvability. Likewise our discussions of free boundary problems were enhanced by this relationship. In 1952, Lewy introduced an elegant way to study boundary behavior of elliptic equations in two dimensions, which is related to free boundary questions. One consequence is that when a harmonic function and a solution of the reduced wave equation in the upper half-plane share Cauchy data on the real axis, that data must be analytic. With trickery and more recently developed high powered theory, and with Joel Spruck, we reproduced this result.

Louis loved movies. One of his favorites early in our relationship, in the mid 1970s, which we saw again together, was *To Be or Not to Be*, the Ernst Lubitsch black comedy with the fabulous Carole Lombard and Jack Benny. Later, when he was home, we talked frequently of books. He read Camilleri and Elena Ferrante. I mentioned *An Odyssey* by Daniel Mendelsohn and Louis had read Mendelsohn's earlier work. To know Louis was to eat with Louis, anywhere in the world. Nothing on his plate went unshared. His collection of stories was infinite and revealing. How can we forget the story of the cassoulet; to retell another time.

He cared deeply for us all. He spoke of Peter Lax and Guido Stampacchia as his brothers. His greatest loss was Cathleen Morawetz. Louis was an exquisite mentor. He inspired us to nurture young people and to offer them vision with warmth and humanity. Remembering Louis is, for us, a blessing.

Luis Caffarelli

I entered the University of Buenos Aires in 1967. The military led the country at that time and many of the top scientists were abroad. Still, the remaining faculty and

Luis Caffarelli is the Sid W. Richardson Foundation Regents Chair in Mathematics No. 1 Professor at the University of Texas at Austin. His email address is caffarell@math.utexas.edu.

Figure 13. In a mathematical discussion with Luis Caffarelli. Courant Institute, 1985.

students at the university were very dedicated. I graduated in Argentina from the Calderon-Zygmund School of Real Analysis and obtained a fellowship to the US. I chose Minneapolis where my thesis advisor Calixto Calderon had gone and arrived in Minneapolis in January 1972. At that time, Minnesota had an exceptional group in harmonic analysis and PDEs, including Aronson, Littman, Weinberger, Serrin, and Fabes; and I also had the opportunity to meet other great mathematicians like Friedman, Lewy, and Stampacchia.

From them I got a deep understanding of PDEs and their applications, and they guided me in their knowledge of new ideas. In one of his visits, Hans Lewy suggested I look at the obstacle problem, where he had done early fundamental work with Stampacchia. It overlapped with the work of Kinderlehrer, a very good friend and colleague in Minnesota at the time who was working in collaboration with Nirenberg. I started to exchange ideas with them, particularly with Louis, through mail, at meetings, and in occasional visits (there was no internet at the time).

In the spring of 1980 while I was visiting Pavia, I received a call from Louis inviting me to stop by Courant on my way back. It was my first time in Manhattan and a very warm reception: Louis, of course, took me to the theater, some museums and, of course, Chinatown where with time I would develop his taste for Chinese food. During the visit, Louis offered me a Professorship at Courant. It was a fundamental advance for me.

The Mentor

Minnesota was an excellent place, but Courant was (is), in the area of analysis, its ramifications and applications, an extraordinary place with all the levels and areas intertwined around the likes of Morawetz, Lax, Varadhan, and of course, Louis. Among the younger ones were Papanicolaou, Klainerman, Kohn, and visitors like Spruck and Gidas, all sharing ideas at noon around the lunch table.

Louis, in particular, created an atmosphere of open discussion and deep creativity, where understanding and advancement was achieved by an open exchange of ideas. The wider impact of Louis on all of us was through his generosity, his love of science, and his sense of community.

Indeed, it is enough to look at his genealogy list of students and descendants or his many collaborators to realize how generous and influential his trajectory has been, how deep is the mark he has left for many generations to come, and how his memory is an enticement for us to follow in his footsteps.

The Scientist

Needless to say, Louis was a deep scientist. He made fundamental contributions from geometry to applied mathematics. A remarkable aspect of his work is how often from a basic property, in a simple context, Louis discovered and developed an extended fundamental theory.

Figure 14. Robert V. Kohn, Louis Nirenberg, and Luis Caffarelli in 2015, about a year after receiving the Leroy P. Steele Prize for their work on the Navier-Stokes equations.

The Agmon, Douglas, Nirenberg contributions are examples, as are the collaborations with Brezis, with Berestycki, with Kinderlehrer, with Yanyan Li, and many others.

I had the privilege of working with him, often as part of a group and it was always a remarkable interaction, where A chatting with B over a cup of coffee, had some vague ideas or doubts and they went and asked about some possible facts to C (often Louis), who often

had the answer and further realized that it was part of a wider theory and enticed us to pursue it. It was a very exciting way of bonding and working.

The Friend

Anyone who had some personal relation with Louis felt his congeniality and warmth. Louis and Sue (and later Nanette) were wonderful companions, enjoying a walk or

a trip or a (great) dinner or any other activity. Furthermore, Louis was always concerned for a friend or colleague's well-being, maybe his health, or being mistreated in his country for political reasons, or just not being properly recognized by his institution. Louis did all that was possible to help.

His loss is of course saddening, but he leaves among us all a trail of exciting science, warm friendship, and happy memories. For all that we are deeply grateful.

Sergiu Klainerman

It is hard to say something about Louis which was not already said. Great mathematician, enormous creative energy, great sense of humor, love for intellectual pursuits of all types, etc. I prefer to tell a short story about my interaction with him at a very difficult personal moment.

I met Louis in the fall of 1976, at the beginning of my second year as a graduate student at the Courant Institute. For me this was a dream come true. I knew about him from some of his papers I had read as an undergraduate in Romania. When I told him I wanted to be his student, he kindly accepted. As an advisor Louis was extremely professional. He would listen intently to what I had to say, make a few comments and suggestions after which, being a very busy person, he would look at his watch as a sign that our meeting was over. That forced me to be very prepared to keep his attention focused and delay that dreadful "looking at his watch moment" as much as possible. With time, Louis became more and more warm towards me, inviting me to his home and taking a fatherly interest in my personal situation.

I had the great fortune to get a problem from Louis, originating in the work of Fritz John, and to prove a result that Fritz found surprising. It required a very technical proof, based on using the Nash-Moser technique in a rather unusual situation. After various presentations in front of Louis, it all seemed correct, though I remember having this strange feeling that I might find a mistake at any moment. And, alas, that moment came at a very inopportune time, a few days before I was supposed to talk about the result in the famous Analysis Seminar of the Courant Institute in front of an expert audience including F. John, J. Moser, P. Lax, C. Morawetz, and even L. Hörmander, who was spending the year at the IAS. On Friday afternoon, less than a week before the seminar, I asked Louis to listen to my rehearsal of the talk and give me his advice. Everything went fine until the moment Louis asked something that seemed like an absolutely routine question. I remember the strange feeling of freezing in place as I instantly

Sergiu Klainerman is a Eugene Higgins Professor of Mathematics at Princeton University. His email address is seri@math.princeton.edu.

realized that I had forgotten to check some properties of the norms I introduced, relative to smoothing operators, in my Nash-Moser scheme. The entire proof seemed to collapse in that moment. I spent the rest of that afternoon trying to convince Louis that my proof was dead. He was very concerned and invited me to his house the next morning to see if anything could be repaired. It is in moments like these when you can really take the full measure of a man. It is hard to describe how supportive Louis was at a time when I was maximally despondent. After convincing Louis that my proof could not be resurrected I spent the rest of the day with him and his wife, Sue, both doing their best to cheer me up. Louis was at his best with jokes and funny stories. He also invited me to a piano concert at Carnegie Hall that evening under the pretext that Sue did not feel well so I could use her ticket. All through the concert I was, of course, still thinking about my problem. It occurred to me that one simple change in the way the Nash-Moser iteration was implemented could get rid of the difficulty with my norms. The concert did its magic. Back home after the concert, I checked the idea and, to my great relief, it worked. I called Louis first thing next morning and he was even more elated than me.

Louis Nirenberg lived a meaningful, creative, blessed life and left a wonderful legacy behind him. We are all better for it.

Wei-Ming Ni

During my college days back in National Taiwan University, one subject I never touched was PDEs. The first year after I came to NYU-Courant, I took PDEs, and was just fascinated by its beauty. My initial adviser, Professor Fritz John, told me to go to Louis, and I did. To my surprise, this great master didn't even ask me any questions (as a "test" of my background knowledge) before he agreed to take me as a student. Years later, when some young student came to me with glitter sparkling in their eyes, I would simply start preparing them for research, as I realized that interest, enthusiasm, and dreams are perhaps among the most important qualities in pursuing truth and beauty in nature.

As a graduate student I had my office next to that of Louis. The seven cabinets of preprints and reprints in my office was his treasure. In contrast to today's USB, it seems difficult for the young generations to visualize how "inconvenient" it was to search for relevant work on a specific topic, and yet, great research was done in those "difficult"

Wei-Ming Ni is a professor of mathematics at the University of Minnesota and a Presidential Chair Professor of Mathematics at The Chinese University of Hong Kong, Shenzhen. His email addresses are wmni@umn.edu and niweiming@cuhk.edu.cn.

old days. Over a two-year period, I had the good fortune to work closely with Louis, learning how he approached a problem, and how he tolerated my stubbornness. Incidentally, our joint paper on the spherical symmetry of positive solutions to elliptic equations was a product of that time.

In summer 2002, I spent a week or so with Louis and Nanette attending a conference in Hanoi, and we took an excursion to the nearby Ha Long Bay. There were many cultural sites and historical buildings along the way with rather detailed descriptions carved in stones at those places—but in Chinese, so even the two tour guides could not understand them, as Vietnam changed its written language completely after its independence. This seems unfortunate as the country literally cut itself off from its long heritage. I had to serve as the tour guide for the entire group. As always, Louis was very interested in understanding more of the culture through historical remains, and Nanette was also very interested, as she was actually born in Hanoi. It was also that summer when my son David had an internship in New York City, and Louis was as kind as always—allowing David to stay in his apartment.

In spring 2003 I was on sabbatical, and I decided to spend one month in New York visiting Fanghua. I was using Louis' office 802, which brought back many fond memories. Nanette was in Paris at that time. Louis and I did many things together—having dinner in the Village, going to the opera in midtown, ... I still remember, after the show we often walked back to his apartment at 82nd Street along Broadway, and we stopped at Zabar's on the way to pick up some delicacies, ... I had never felt so close to Louis.

The last time I visited Louis was in the fall of 2017, with Yanyan. Louis was about to go home from Village Cares—a rehabilitation facility on Houston Street. We brought Louis his favorite fish-dumplings and shrimp-dumplings from his favorite restaurant in Chinatown. Every time he saw me, Louis always updated me with the latest news from NYU-Courant, and he always wanted me to tell him what I was working on mathematically. My interests had gradually turned to mathematical ecology by then, and he was surprised by what mathematics could do in ecology.

Louis was the single most important person in my career, and a friend close to my heart in my life.

Ivar Ekeland

Louis taught me the answer to the question we keep asking ourselves: "Why do you do mathematics?" "For the grudging admiration of a few friends." This was typical

Ivar Ekeland is an emeritus professor of mathematics at Université Paris-Dauphine. His email address is ekeland@ceremade.dauphine.fr.

Figure 15. Left to right: Fang-Hua Lin, Ivar Ekeland, Louis Nirenberg. Pittsburgh, 2005.

of his sense of humor, steeped in the tradition of Yiddish jokes and borrowing from his experience of mathematicians. His jokes were always fun, never at the expense of others, and delivered truth in a self-deprecating way. This one was no exception.

No one has made more friends than Louis by doing mathematics. With very few exceptions, all his papers, including the major ones, arose from collaborations, and the collaborators became friends. He had infinitely many students, and he kept in touch with them. The result was a dense network of friends and admirers, almost a family, stretched all over the world around Louis and Nanette, which was a great comfort to him in later years.

He was intellectually very generous, more interested in what others were doing than in what he himself had done. I remember Louis, at the 2015 Abel Prize ceremony, charging up to the podium in his wheelchair, and taking the audience by storm. He was honored, he said, to have received the prize, but even more to share it with John Nash, whose fundamental work had inspired him since his early days, and so on and so forth. There was Louis, full of enthusiasm, raising the audience, not for himself, but for someone else! He was totally sincere about it. He loved mathematics, no matter how much he knew he wanted to know more, and if someone else found a good result, then here was a golden opportunity to learn! He went into the proofs with pen in hand, and could extract ideas and information from the most minute details.

Louis was a man of culture. He read, enjoyed music, and loved the movies. During my stay at Courant, New York movie houses still had “double features,” two movies

in a row. He went with me, and if he could not go, he sent me, and I had to report the next day. This is how I discovered Kurosawa: he took me to *Ikiru*, that absolute masterpiece, and we went together to the premiere of *Ran* in New York. He was kind and warm; his house was always open, in New York as in Paris, with Sue as with Nanette. Life held a special taste for him, blending European culture, the warmth of friendship, and the drive to know. He died without having exhausted what it could bring him. But, to quote another of his jokes (the waiter’s epitaph): “God finally caught his eye.”

Kung Ching Chang

I was not an official student of Professor Nirenberg, but a visiting scholar at the Courant Institute of Mathematical Sciences under his supervision at the end of the 1970s. While there, I listened to his class “Topics in Nonlinear Functional Analysis,” in which he naturally bridged the gap between the abstract theoretical approach and works devoted to the study of specific equations. His style was very different from any book I had read before. It greatly changed my approach to doing mathematics. I have benefited a lot from his teaching. He also gave me some new preprints to read, and as a result some new ideas flashed into my mind. Under his meticulous guidance, patient help, and warm encouragement, I renewed my knowledge and caught up with new trends at the frontier of mathematical research. Probably because I came from an isolated environment, he took excellent care of me.

Louis was very personable and was very fond of movies and art museums. When I was at NYU, I used to meet him once every few weeks. After discussing mathematics, he would always recommend some movies and exhibitions to me. He would take out a newspaper and mark in front of the titles a few stars as assignments for me. Once, when I left for Italy, he recommended many churches and museums. Among them he strongly recommended Galleria degli Uffizi, saying “You have to go!” Chinese film was entering the international stage in the eighties, and he once asked me for a recommendation. I suggested to him the movie *Red Sorghum*. Later I found out, he not only watched it but also watched the movie *Ju Dou*, made by the same director and the same actress afterwards. He was full of praise of the latter.

Professor Nirenberg loved China. He supervised many Chinese graduate students and collaborated with many Chinese mathematicians over the years. He visited Beijing in 1975, 1980, 1988, and 2000. Early on, he predicted the

Kung Ching Chang is an emeritus professor of mathematics at Peking University. His email address is kcchang@math.pku.edu.cn.

Figure 16. Left to right: Louis Nirenberg, Gabriella Tarantello, Kung Ching Chang. Cambridge, 2000.

increased international recognition of Chinese mathematics. He accepted an honorary professorship from Peking University in 2016, when he was over 90 years old. He did intend to visit Beijing again, but due to health reasons, the travel wasn't possible.

We maintained our friendship for more than 40 years. From 1980 to 2020, the swap of New Year's cards was never interrupted. In the beginning, the cards were made of paper, then they became e-cards, and in recent years, due to his health conditions, they were emails in which we interchanged our status with regard to health and daily life. His mail always came around Christmas time, but in 2019, it was late. I felt that was an ominous warning that his cancer had spread. On January 8, 2020, I finally received his email, by which I could imagine how difficult it was for him to type. This was the last communication I received from him.

Professor Nirenberg was not only a true master of mathematics, but also a man of high moral character admired by our entire community.

Sun-Yung Alice Chang

It is hard to accept that Louis Nirenberg has passed away—he has been in our thoughts and frequently in our family conversation since then.

The relation of both Paul Yang (my husband) and I with Louis started in our early careers—we were educated in graduate school and early career through his writing and his lecture notes. He always made the ideas of mathematics so natural and beautiful and (at least in appearance) easy to understand, which heavily influenced our view and our love of the subject—geometric analysis and analysis in general.

One problem which strongly influenced my early career—and figures also in my most recent work—is a question Nirenberg posed around 1970. His penetrating

question was “what are the functions which can be the Gaussian curvature function for some metric on the 2-sphere?” This question echoed strongly with the later development of “prescribing scalar curvature on the n -sphere S^n when $n \geq 3$.” On a general manifold, when the prescribed scalar curvature is the constant function in a fixed conformal class, this is the famous “Yamabe problem,” solved eventually by Yamabe-Trudinger-Aubin-Schoen in the 1980s. The work of Louis with Haim Brezis in the 80s on elliptic problems with critical exponents was motivated in part by the Yamabe problem, and it opened a whole new research area. In 2016, in celebration of Louis Nirenberg as a recipient of the 2015 Abel Prize, an article in the *Notices* of the AMS discussed some of his work and its later consequences [3]. I contributed a section, giving an expository account of the influence that Nirenberg's question from 1970 had upon the field of geometric analysis.

I remember vividly how around 1982, as a young mathematician at the beginning of my career, without any previous personal connection, I nervously contacted Louis and asked for an appointment to present some work Paul and I had done on his problem—the main idea was to play with the constant in a sharp inequality of Moser-Onofri, which allowed us to tackle the problem of prescribing the Gaussian curvature function without imposing some extra “symmetry” condition on the function (e.g., when it is an even function) and which eventually led to an index count formula for the problem. Louis gave us a generous period of time and was gracious in listening during our visit. To our surprise, he asked us to join him for lunch the next day; and during the lunch, asked us a number of follow-up questions. Later on he informed us that he was impressed by our work and thought we had penetrated the problem. This praise, coming from Louis, was such a strong encouragement for us!

Later on in my career, whenever a young person asks to talk to me presenting his/her work, I always remembered how I was treated and do my best to arrange the meeting.

We later had frequent contacts with Louis, at Courant, at various conferences (in particular in Italy, his favorite country), and later on more often by visiting him and Nanette at his apartment at New York. Each time he was eager to know what mathematical problems we were working on, and we felt it an honor to have the chance to present to him the ideas we had and the stumbling blocks we faced. His view was always so global, and his mind so sharp, that we really benefited from the discussions. Our meetings usually ended with casual chatting and dinner at some fantastic local restaurants. Louis knew so much about food—I would say his knowledge of different types of Chinese cuisine far exceeded mine and most of my friends. He was curious about everything in life—he once

Sun-Yung Alice Chang is a Eugene Higgins Professor of Mathematics at Princeton University. Her email address is chang@math.princeton.edu.

asked me why the leaves of the tea he drank in Taiwan would open up after pouring in hot water, something I grew up seeing everyday without ever pondering!

Time spent with him was like the Chinese saying, you feel that you are “in the gentle breeze in the Spring” (如沐春風). He was such a great mathematician, magnificent teacher, and wonderful human being. One seldom meets such a person in one’s life. Paul and I feel privileged to have come under his influence and we truly miss him.

Jalal Shatah

I was fortunate enough to meet Louis during the first week of my postdoc years at Courant in 1983. We quickly developed a friendship that lasted until his recent death. My daughter, when she was 5, told me that since she did not have a living grandfather that she would like to adopt Louis as a grandfather. Louis was very flattered and thrilled by this, and commented that we were always close like family.

When I first met Louis, the thing that struck me was his kindness and humility. I was expecting the icon of PDEs to take the role of a master, but instead he took the role of a friend. His kindness impacted me both mathematically and personally. I recall very early on Louis inviting me to dinner (the first of many) where I met his family, Sue, Lisa, Marc, and later his sister Debby.

Louis was always particularly kind to young mathematicians. He did his best to promote their work. I was one of those when in the summer of 1987 Louis decided to give a talk at Collège de France about my work. The auditorium was packed when Louis stood up to talk, but instead of talking about his own work he spoke about my work on normal forms and my joint work with Henry McKean on conjugation of flows. I have also seen this happen on other occasions with Louis. He did it during talks and at lunch at Courant.

Central to our friendship was a shared love of movies, world music, and good food and wine. Louis was always able to find good things to say about a movie, even when it was bad. When we both agreed that a movie was bad he would say something like “but there were a couple of shots that were original.” This was Louis’ nature. He could always say nice things even about mediocre stuff. He could at the same time be brutally honest. He was no shrinking violet. By the way Louis had a wicked sense of humor, enjoyed telling jokes, many of which he attributed to Peter (Lax).

Jalal Shatah is a Silver Professor of Mathematics at the Courant Institute of Mathematical Sciences, New York University. His email address is shatah@cims.nyu.edu.

Figure 17. With Jalal Shatah, 1991.

As for food, Louis was the master of enjoying a good meal. For many years Louis, Raghu (Varadhan), Henry (McKean), Ray (Michalek), and I with our families together sampled restaurants reviewed in the *New York Times* (NYT). Louis always brought with him a clipping of the review and ordered what was recommended, and throughout the meal we compared what we liked to what was written in the review. These were delightful years during which we discovered many gems in NYC. These outings lasted until November of 2019.

Our last two get-togethers occurred in December 2019 and January 2020. After having dinner at his place in December, and while walking out, I noticed a bunch of math papers spread out on his dining table. When I inquired about that, Nanette (Aubin) quickly responded that he had been working all day long on a problem. I asked him about the problem and he explained that he was working on the nonexistence of stationary solutions of the Navier-Stokes equations with finite energy. I was amazed that after all that Louis went through physically, he still had a sharp mind and the drive to spend the whole day working.

My last visit to see Louis was in mid-January. We chatted about Courant, our families, and we started making plans and deciding which restaurant to pick to celebrate his 95th birthday the following month. Alas he died shortly before. My wife Deborah commented that New York does not feel the same without him. It lost part of its attractiveness for me.

Fang-Hua Lin

I joined the faculty of the Courant Institute in 1985. It is striking to realize that when I joined the Institute, Louis was about my current age. Though I was then in my

Fang-Hua Lin is a Silver Professor of Mathematics at the Courant Institute of Mathematical Sciences, New York University. His email address is linf@cims.nyu.edu.

20s, it never crossed my mind that he was in his 60s; I thought not about his age but about his energy. During the late 1970s and early 1980s, some of his most cited mathematical papers of great importance and remarkable influence were published: Gidas-Nirenberg on the moving plane method to study the monotonicity and symmetry properties of solutions of elliptic equations; Caffarelli-Kohn-Nirenberg on the existence and partial regularity of suitable weak solutions of the Navier-Stokes equations; Kinderlehrer-Nirenberg on the smoothness and analyticity of free boundaries; Brezis-Nirenberg on nonlinear elliptic equations involving critical Sobolev exponents; and a long series of papers with Caffarelli and Spruck (one with J. J. Kohn as well) on the Dirichlet problem for fully nonlinear elliptic equations, including the real and complex Monge-Ampère equations. When I got to Courant, Louis had just completed work with H. Berestycki and Raghu Varadhan on the sliding method—a new maximum-principle-based approach to studying the monotonicity and stability properties of traveling waves.

I feel very fortunate to have been Louis' colleague, and it has certainly helped my career. A special advantage that I have enjoyed is that my office has always been on the same floor as his, and I had permission to knock freely at his door. Through his suggestion I got a chance to read many classical articles, and sometimes even material that may never have been published. Once he dug out a manually typed manuscript by E. Hopf, on which there were handwritten comments by both Louis and Hopf. One of the theorems I read there is, I believe, still not widely known. As a junior colleague of Louis, I constantly experienced his generosity, care, and love. He was simply a great human being.

As anyone who has communicated mathematics with Louis knows, he was especially gifted at catching mistakes or gaps in "proofs." Therefore, it was often the case that when I came up with a new thought, I would discuss it first with Louis. If he showed hesitation or was puzzled about some aspect of the argument, you could be pretty sure there was a mistake. Sometimes, I would be really excited to tell him about the solution to a problem I had been thinking about for a long time, and then at the end of our conversation Louis would point out a step that seemed to have a flaw. It could be rather discouraging. But Louis would always find a way to offer encouragement and comfort. At one such time he said "...I must tell you about a mathematical review that I once saw. It read something like this: the author uses very heavy machinery and provides a very long complex and intriguing proof of something which is obvious. However, there is a non-correctable mistake at a key step of the proof, and the mistake is not new either." Louis did start to complain

Figure 18. S. R. Srinivasa Varadhan, Louis Nirenberg, Henri Berestycki. International Conference on Partial Differential Equations in Celebration of the Seventy-fifth Birthday of Louis Nirenberg, Taiwan, September 2000.

about 20 years ago that he was no longer as good as he used to be at finding gaps in proofs. I thought then: he was still pretty good at it, considering his age!

Louis Nirenberg passed away recently. I miss him! "If they ever tell our story, let them say that we were colleagues and protégés of giants." He truly was a giant!

Henri Berestycki

There are mathematicians who, to achieve a great result, make an immense, quasi-athletic effort and succeed by their remarkable stamina; there are those who as ascetic hermits, devoted to the cause of mathematical knowledge, are capable of extraordinary and admirable efforts; some who even have to suffer to reach their goal. And then, there are mathematicians who see their work as one of the truly enjoyable human endeavors. Louis Nirenberg was one of these. To me, he epitomized the joy of doing mathematics.

All mathematicians enjoy their discoveries and are happy to talk about their work. What I have in mind is the attitude during the process of research, the path that leads to the results, the days of effort, the nights when nothing converges, yesterday's seemingly tight proofs that are shattered the next morning. It is about the anxious doubt of whether the long and complex calculations will lead somewhere, worrying that, along the way, one might have forgotten something, made a tiny and unremarkable error that will threaten the whole edifice. Louis enjoyed and loved it all! But then, of course, he made few errors and his attempts were often successful.

This extraordinary aspect of his personality was contagious. It made the daily experience of working with him an intense pleasure. He was a seemingly soft driving force,

Henri Berestycki is a professor at École des Hautes Études en Sciences Sociales (EHESS), Paris. His email address is hb@ehess.fr.

not imposing but instead inviting to share a great pleasure. Even if most of the time we were impressed by the towering scientist, he was always warmly welcoming and encouraging.

From the mid 1980s to the mid 1990s Louis and I collaborated intensely and published 17 articles. The topics range from models in combustion theory to traveling waves, the moving plane method, eigenvalue problems, the qualitative theory of elliptic and parabolic nonlinear equations, and, of course, the maximum principle that serves as a common thread. He often commented on the twists and facets of the maximum principles for elliptic and parabolic PDEs and on how much you could still explore about it and draw from it. He liked to say “I have made a living out of the maximum principle.”

When I was visiting the Courant Institute then, the days followed a routine. We started by reporting on the previous night’s findings (or on what we had gleaned on the subway to the Institute). After working in the morning, we would go for lunch and a special treat was to go to Chinatown. I remember in particular the restaurant Great NY Noodletown, where Louis was treated with extreme respect as “the professor” (somehow, they had learned what an important mathematician he was). Then, we would stroll back through little Italy, stopping at a coffee house en route to the Institute. Or, instead of Chinatown, we would occasionally go to the Second Avenue Deli for a generous helping of pastrami (although the blissful lethargy it induced could lead to some of the difficulties in convergence and errors I mentioned).

Mathematical discussions with Louis were enlightening and not just for the outpouring of great ideas. He shared all kinds of methodological insights on how to conduct research. He once explained: “when you want to prove something, you should look for a counterexample. If you find one you know you were wrong, and if you don’t, you gradually understand what it is that makes it work.” I vividly remember the Sherlock Holmesian “aha!” at the whiteboard when something became clear, or when he would approve of a suggestion. Also, the unmistakable look, from above his spectacles, when he was dubious of or surprised by what you said, or found something fun.

He was a great practitioner of the art of conversation. Our mathematical discussions were always intertwined with other topics. We could start talking about our projects and often at some point, we would drift into more mundane subjects, such as books, movies, politics, restaurants, music, the French, Italy ... all topics very important for him. We often diverged, especially on movies, and he accused me of “French formalism.” But it was great fun and stimulating to talk with him, advancing viewpoints, and risking, here too, to get his famous glance from above the

spectacles. And then, at some point, all of sudden, in the middle of something completely different, he would say: “I have an idea” and we would get back to our topic of the day. Mathematics with Louis was interwoven in the fabric of life.

Louis’ charisma was gentle and seemingly subdued (except when ordering in restaurants). Yet this made it all the stronger not only in mathematics but also in life in general. He never imposed, but you felt compelled by his example. He was a man of very strong principles and Louis was very much inspiring in this respect as well.

His lectures were always a treat. Invariably witty, fun, and beautiful, they were fascinating and stimulating. They were also deep and generous—they allowed you to get a sense of his vision. Even when I heard him talk about our joint work, not only was it fascinating to hear how he would stage the problems and mathematical developments, but I could always also glean something new and inspiring, like unexpected connections between things. Later in life, lecturing from a wheelchair, he was still able to captivate audiences in a unique manner. Louis’ mind remained sharp until the end. A few years ago, as his health declined, he wrote me in an email: “I am still working on some problems, and everything I try fails, but it is still fun to try.”

Gang Tian

I first heard about Louis Nirenberg from my former adviser, K. C. Chang, at Peking University in early 1982. Professor Chang had just come back from the USA where he visited the Courant Institute of Mathematical Sciences and became a friend of Louis. We studied Louis’ work on nonlinear analysis. In 1983, we read his breakthrough joint paper with L. Caffarelli, J. Kohn, and J. Spruck on Monge-Ampère equations. I

was immediately attracted to Monge-Ampère equations and proved a new estimate of high order as a part of my Masters degree thesis at PKU. His work has influenced me a lot in my later work which involves Monge-Ampère equations.

Figure 19. Louis Nirenberg accepts an Honorary Professorship from Peking University. Left to right: Yanyan Li, Louis Nirenberg, Gang Tian. Manhattan, 2016.

Gang Tian is a chair professor of mathematics at Peking University, and an emeritus professor of mathematics at Princeton University. His email address is gtian@math.pku.edu.cn.

I first met Louis in the spring of 1990 when he was giving a lecture at Princeton University. We chatted and he invited me to visit the Courant Institute. I felt very honored and happily accepted. A year later, I took a job offer from the Courant Institute. I think that it was Louis who initiated the process of bringing me to Courant. I always felt fortunate to become a member of the Courant family. My stay at Courant was one of my best times in my mathematical career, where I established a mathematical theory for quantum cohomology and introduced the notion of K-stability.

When I was working at Courant, Louis and I often met to talk about mathematics and other things. I learned a lot from him on mathematics, history, restaurants, and museums. He was always patient to listen to my new research work, even on some topics which were not close to his research interests, and gave very good advice. When I proved that the existence of Kähler-Einstein metrics implies the K-stability, I gave him a detailed presentation on its proof which uses complex Monge-Ampère equations. He told me some history on the study of complex Monge-Ampère equations and commented that the proof was not standard. This result was published in *Inventiones* and became my most cited work so far. I am very grateful for his generous support during my career, from which I continue to benefit.

I also learned many things from him besides mathematics. One day in early 1995, a few friends gathered with Louis to celebrate his 70th birthday. After dinner, we went to his home in New York City for drinks. He brought out a bottle of Grappa from 1904. This was the first time I tried Grappa, a very exceptional one, and learned that it is an Italian hard liquor. Since then, I have often had a glass of Grappa after an Italian dinner.

In 2016, because of his outstanding contributions, Peking University offered him an honorary professorship which he accepted. I had the honor of presenting the certificate to him in New York City. So his passing is not only a loss for the Courant family but also for Peking University and the whole mathematical community.

Gabriella Tarantello

It was a quiet Sunday morning when the phone rang. I was working and taking advantage of that calm moment, to check the details of a geometrical inequality I came across while learning about Teichmüller's theory of Riemann surfaces. I knew with whom I was going to discuss it first. I anticipated a nice conversation over the phone, getting

Gabriella Tarantello is a professor of mathematics at University of Rome "Tor Vergata." Her email address is tarantel@xp.mat.uniroma2.it.

his (always needed) encouragement and the comfort of his sheer happiness to see me busy and involved in a new research project.

But Louis Nirenberg had passed away a few hours earlier, so I learnt from the colleague who had just called.

My first thought went to Nanette, Louis' lady friend, in mourning alone in Paris. Louis had been waiting with moving eagerness to see her again in New York in a few weeks.

Then I was overwhelmed with a thousand memories.

It made me smile to recall Louis insisting on the fact that he had been only a lucky (rather than exceptional) mathematician. Actually, the lucky ones were all of us, the great family of students, collaborators, and colleagues who could always count on his exquisite friendship, availability, and generosity.

Louis Nirenberg loved Italy. So when I had to make the crucial decision to move back he provided a thousand arguments why I should not hesitate. But really what I could not ignore was that every time Louis came back from a trip to Italy he was rejuvenated and in splendid spirits. Always, he would come back with fantastic stories to tell. And indeed he was an extraordinarily funny story teller. For example, I can still see him describing the struggle to acquire a beautiful set of Italian ceramics from a skilled artist (also described as an extremely good seller) who would not bend over the high price. But Louis could not resist such a challenge and finally got the ceramics after an incredibly long but successful negotiation.

There is no doubt that learning and discussing mathematics with Louis Nirenberg has forged my style as a mathematician. I admired the collaborative atmosphere that Louis helped to create among his students and collaborators. There is a strong bond that keeps us close and connected (mathematically and otherwise) in a very special way. From Louis' teaching we acquired a certain taste for a "good problem" or a "nice proof," which makes us strive to reach far beyond the mere technicalities. Still we prized Louis' great ability to carry out involved technical arguments with elegance till the last detail.

I had only to knock on the door connecting our adjacent offices to have the privilege to learn more about many beautiful inequalities and estimates and the secrets of their making. Louis was always available for discussions with us students.

To encourage us in those moments when nothing seemed to work, Louis would say that a good mathematician can hope to have (on average) at most two original ideas in a year. This, translated into the current lingo, means at most two publications per year. Perhaps it would be appropriate to reaffirm this principle with the younger generations.

Louis loved to eat well and in the company of dear friends. Chinese cuisine was his favorite. We had a thousand lunches and dinners (often in Chinatown) in search of the “best” Chinese restaurant in town, following the suggestions and in the company of Louis’ many students from China. So when I finally confessed to him that my favorite cuisine was Japanese, he smiled, saying: “nobody is perfect!”

Instead, we shared the same passion for movies and enjoyed going to the cinema. In one of the endless discussions about the last film we had seen (our tastes often diverged) I happened to tell him that in my childhood I spent every Sunday afternoon in the Cinema Hall (held by the parish) of the small village of Abruzzo where I grew up, viewing the same film over and over again. Louis’ comment on this story was: “it just proves that you were a born mathematician!”

Louis’ humor is well known. Amazingly he had a good joke to tell on any occasion. That includes the very last time I saw him in New York, when he had yet another Yiddish joke to tell. It was so funny I had tears in my eyes after the big laughter. Different are the tears in my eyes now, while I recall those unforgettable marking moments of my life.

I’m not going to talk about Louis’ deep and elegant mathematical results, which have nourished generations of analysts over the years and all over the world. For them Louis has received many prestigious awards. I only wish to recall that together with Louis and Nanette we watched (at a hotel in Rome) the video of the Abel Award. This is my favorite video of Louis. He is irresistible in just being himself—overwhelmingly funny, sweet, witty, and (above all) a nice human being. I was so terribly proud of him and finally I found the courage to tell him right then.

Yanyan Li

Louis was my PhD thesis advisor from 1984 to 1988. It was my good fortune and privilege to be able to meet him regularly and work with him closely. He was an exceptional mathematician. He influenced my way of thinking and taught me much mathematics. Over time, Louis and I developed a father-and-son relationship. On several occasions, he introduced me to his friends saying he regarded me as his son, and I also looked up to him as a father figure. Louis had a significant personal impact on my life and the world is very different for me without him.

Louis was superb in guiding his students. He left enough time and space for students, and always sought to

Yanyan Li is a Distinguished Professor of Mathematics at Rutgers University. His email address is yyli@math.rutgers.edu.

Figure 20. Left to right: Yanyan Li, Louis Nirenberg, Marjorie Lee, Ray Michalek, Nanette Aubin. Flushing, 2006.

encourage and support them. In my case, Louis suggested problems for me to work on and papers to read, which led me to discover problems that were to my interest. Having the office adjacent to his gave me access to the abundant file cabinets filled with papers he had kept. Furthermore, Louis had many distinguished visitors with whom I was able to talk (and often also share a meal). This exposed me to the state of the art in various research fields, and created lifelong friendships.

The first problem Louis suggested to me was to define an integer-valued degree for second order fully nonlinear elliptic operators. A mod 2 degree for proper Fredholm maps of index zero was defined by S. Smale in 1965. One day, after working on it for quite some time without success, I presented Louis a number of attempts I had tried. Then Louis went to the board and made attacks on the problem with long computations. Within a month of that day, I successfully solved the problem. Apparently, Louis had put me on the right track without me even knowing!

During the period from 1998 through 2013, Louis and I published 13 papers on a number of subjects including elliptic systems describing composite materials, Finsler metrics and Hamilton-Jacobi equations, extension of a result of A. D. Alexandrov on embedded constant mean curvature hypersurfaces, and, joint with Luis Caffarelli, on singular solutions of nonlinear elliptic equations. Louis and I met regularly and discussed mathematics until December 2019. Most of our mathematical discussions were at his home. For many years, we used to start a routine working day with a late breakfast of Danish pastry and espresso from Zabbar’s, then we worked on mathematics. In between mathematical discussions, we used to have lunch in restaurants nearby. We also had many mathematical discussions when our trajectories to international conferences or visits to universities overlapped. Among those conferences there were about ten held in his honor. How much Louis was loved and respected among mathematicians worldwide!

Talking with Louis was great. His wisdom went far beyond mathematics, and he also had wonderful jokes on almost every occasion. He had a positive and uplifting

attitude towards life, and each time after meeting him the world seemed brighter to me, making me happier and more energetic.

Lastly, Louis loved delicious food. My wife Marjorie and I regularly had dinners with him, along with Sue or, later, Nanette, Louis' sister Debby, and Ray (Michalek). Louis was very passionate about discovering new restaurants and trying them out with my family, and we always had a pleasant time. We sampled a variety of cuisines and restaurants — especially many Chinese restaurants, including a number of them in Flushing, which he liked a lot. Most importantly, Louis' consistent optimistic view on life and witty humor will forever be remembered and will continue to have a lasting impact on my life. He lived his life to the fullest and continued to age 94 having a sharp mind that motivated others around him. We all love and miss Louis greatly!

Isabeau Birindelli

I was a PhD student at the Courant Institute 1987–1992, and Louis Nirenberg was my advisor. He was an outstanding advisor—very dedicated to his students. He didn't assign specific problems, and this was a great source of anxiety for us. Rather, he would suggest articles to read—expecting that problems to be solved would emerge from the natural questions one should ask oneself while reading good mathematics. When you went to his office with what you thought was a good but complicated idea, he would come up with a simple counterexample—one that didn't just show you were wrong, but also indicated how to proceed on the right path. He taught me to study simple cases in order to solve more complicated ones. In order to give a hint of Louis as an advisor, I recall that one day, for reasons I can't remember, Louis suggested we work at his home. I loved going there, it reminded me a bit of my family home, with memories of his travels, his beautiful masks, and Sue was extremely welcoming. I was stuck on a problem; I needed to construct a barrier function and the natural candidate was the principal eigenfunction of some operator. He had a whiteboard in his study and I used it to explain that the eigenfunction was “not enough,” that I needed some “forcing term,” modifying the equation on the board accordingly. I said “but then by the maximum principle the solution would have the wrong sign.” He looked at me in silence for a couple of seconds, and I froze. When he looked at you this way there were two possible causes: either you hadn't closed the special whiteboard pen or you just said something wrong. Then I realized that

Isabeau Birindelli is a professor of mathematics at Sapienza Università di Roma. Her email address is isabeau@mat.uniroma1.it.

Figure 21. Left to right: Louis Nirenberg, Peter Lax, Isabeau Birindelli, Mila Spani, Fang-Hua Lin, Jalal Shatah. PhD thesis defense of Isabeau, 1992.

we were not within the hypothesis of the maximum principle! That was good, but left me with a great uncertainty. Suddenly Louis' expression changed and with a slight excitement, that would require a better writer to describe, he suggested “it could very well be that the sign of the eigenfunction propagates above the principal eigenvalue, a sort of anti-maximum principle” I still had to prove it, but by the expression on his face I knew that it would be possible to do it. The anti-maximum principle became one of the main results of my thesis.

The best lunch I had in my life was with him and Henri Berestycki, at Katz's Delicatessen. At the time Louis was working with Berestycki, who often visited the Courant Institute. Louis invited me to lunch, as an occasion for me to talk to Berestycki about my PhD problem. But things turned out completely differently, probably due to the very special atmosphere at Katz's Delicatessen. They started competing in who could be more funny and witty, in a sort of dialectic “duel” that would have certainly been inserted in a movie had Woody Allen been present. I am very sorry not to have recorded it.

Louis Nirenberg was a great mathematician, winner of many of the most important prizes in mathematics. But he also had another great talent, as every one of us that had the privilege to know him can confirm—a talent that doesn't give prizes but gives friends: he had a terrific sense of humor. Louis was always ready with a joke, a story, a witty remark, a punch line, ready to surprise and make you laugh especially since it was told with great seriousness.

Looking at you above his glasses with eyes that sparkled with intelligence, he had the capacity to make you feel welcome and scrutinized at the same time. He used to say that the mathematical community was like a large family which he was happy to meet around the world in all the different conferences and university gatherings. If someone

from this “family” was not present he would send him/her a postcard, having everyone signing it. He loved life in its many aspects, he was fascinated by the feminine world, he was a great lover of movies and of New York City, he was always happy to come to Italy where he appreciated food, art, history and where he had many friends. Until the very last months of his life, whenever someone suggested a good book, some restaurant, a special movie, a good show he would take out his little black book and would take note of the recommendation in his unreadable script. (I hope these notebooks still exist—they must be full of suggestions—though I am not sure anyone would be able to read them.) Curiosity, in the broad sense of the term, kept his mind young and sharp until the end.

Dear Louis: the world is a little less interesting without you, and your mathematical family misses you.

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