Anatole Katok—A Half-Century of Dynamics

Boris Hasselblatt

Katok in 2006.

Anatole Katok (August 9, 1944–April 30, 2018),¹ a leading dynamicist, a Fellow of the American Mathematical Society, and a member of the American Academy of Arts and Sciences, began his career in Moscow, spent six years each at the University of Maryland and the California Institute of Technology, and worked at the Pennsylvania State University for almost three decades. He died in Danville, PA, from pneumonia and complications from an infection. Robert J. Zimmer of the University of Chicago called him “a whirlwind of mathematical activity” who expanded the boundaries of his field and “brought new connections and engaged all around him with an infectious and buoyant en-

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Communicated by Notices Associate Editor Bryna Kra.

¹*Анатолий Борисович Каток in Russian usage.*

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DOI: <https://dx.doi.org/10.1090/noti1883>

thusiasm for mathematics and its mysteries” [32]. He “was an extraordinary man and a great mathematician, one of the giants in dynamical systems and ergodic theory” (Gregory Margulis, Yale), “a singular indomitable force of will” (Michael Boyle, Maryland), and “one of the most inspired, and inspiring, mathematicians of a generation” (Marcelo Viana, IMPA). Indeed, from early in his career Katok stood out for his ability to be involved in essentially all areas of dynamical systems. He always worked in dynamics in a broad sense, and his interests ranged widely, both scientifically and in terms of the character of his publications and activities. Beyond his own large body of work, at the breadth of which his *Annals* papers [1, 5, 7, 13, 15, 18] only hint, he fostered the creation of mathematics through mentorship; the sharing of ideas; and the organization of conference series, journals, and the Center for Dynamics and Geometry at the Pennsylvania State University, which was recently endowed and renamed in his honor.²

Moscow

Anatole Katok, “Tolya” to his friends and family, was born a mere 200 miles from where he last lived, in Washington, DC. His father Boris (a metallurgical engineer) and his mother Dora (a chemist) belonged to a Soviet delegation working with the American lend-lease program. Begun even before the attack on Pearl Harbor, this program helped defeat the Axis powers by “lending” food, oil, and materiel to allied countries, principally the British Empire and the Soviet Union. The latter received some 11 billion USD worth of support (out of over 50 billion total), and this was crucial for the Soviet military effort. Even much later, Katok recalled ancient US vehicles from this program as a significant part of the Soviet automobile landscape. His

²<https://math.psu.edu/dynsys/>, <https://news.psu.edu/story/524919/2018/06/12/administration/donors-honor-memory-penn-state-professor-anatole-katok>.

Katok with his father, Boris.

parents were sent on this mission by the government, but not as diplomats. As a consequence, Katok was a US citizen by birth, a material point in his later biography. And he relished pointing out that he was therefore eligible to be elected president of the United States.

Katok's father ranked high in the Soviet hierarchy, and he died in his early sixties (apparently due to heart problems). Katok was raised principally by his mother, from whose first marriage he had a half-brother, Alexander (Sasha) Gruz, 10 years his senior. In 1975 Sasha and Dora immigrated to the DC area, where she lived to the age of 93. She recognized and cherished Katok's talents early, and he was an acknowledged wunderkind. Among his favorite childhood books was the *Great Soviet Encyclopedia*. He once told me that he had some regrets about the disdain for the practical that came with this admiration for the high feats of his intellect, which left him being "challenged" sometimes in practical or hands-on matters. Indeed, he did not take to technology naturally. In the initial years in Maryland he did not care to even acquire a driver's license—yet soon learned to drive cars with a manual transmission, in the ancient Beetle of Robbie Robinson, his first US doctoral student. He loved cars (his coolest was probably the yellow early 70s Mercedes-Benz convertible with hard and soft tops), though at times their basic functions would require some concentration. In the late 1980s there was also discernible surprise when colleagues first got an email from him. But he was no Luddite: he embraced email, and over time his hammering index fingers would type substantial texts. And he maintained a large and useful web page³ as well as a Google Scholar account.

In the ninth grade, he transferred to Moscow School N59, from where the likes of Arnold and Maslov had graduated, and he intended to enter the Lomonosov Moscow State University before his tenth and final grade.

³Now archived at akatok.s3-website-us-east-1.amazonaws.com.

In 1960, Katok's first year at the university, with Kazhdan (first row, second from the left), Margulis (upper left corner), and Sergey Gelfand (right upper corner, third from the right). Katok liked this photo very much and kept it on his office wall.

With the help of Aleksei Markushevich, the vice-minister of education of the RSFSR, he obtained permission to take the external examination for high schoolers after the ninth grade and hence never needed to graduate from high school. Yulij Ilyashenko recalls a newspaper headline "Markushevich was struck by the mathematical talent of the youth." Katok having been an alumnus of "mathematical circles" for high-school children (in his case supervised by A. Egorov and N. Vasiliev), Katok's next move was to affect his life well beyond his career. Upon entering the university in 1960, he immediately began teaching such a circle himself, and among his students was Svetlana Rosenfeld, three years his junior (and also at Moscow School N59). They fell in love and married as early as legally possible. They were married for 52 years. Their daughter Elena was born in Moscow and is now the Ashbel Smith Professor at the Naveen Jindal School of Management of the University of Texas at Dallas. Their son Boris, also born in Moscow, is a Senior Software Developer and the owner of Coconut Tree Software in Sparks, Nevada. Danya Katok was born in Hollywood and is an acclaimed soprano in New York City. Katok was rightly proud of his wife and children and went out of his way to support them.

Katok started his third year under the supervision of Robert Minlos, but his enthusiasm waned by the end of the year. While Minlos was an inspiring teacher, the problems he assigned in his seminar required a lot of technical skill and bored Katok. Thus, Katok started attending Sinai's special course

Katok circa 1963–1965.

on ergodic theory in 1963, just as Sinai began to receive wide recognition. They started meeting and discussing mathematical problems, and Katok worked with Sinai for his last two and a half years at the university as well as in graduate school.

The theory of dynamical systems (Katok was not fond of the term “chaos theory” used in connection with some applications) is a mathematical discipline that studies group actions—initially, $\mathbb{Z}$ -, $\mathbb{N}$ -, or $\mathbb{R}$ -actions—with a view to describing the long-term behavior in probabilistic, geometric, smooth, and topological terms. With the seminal work of Kolmogorov, Smale, Anosov, and Sinai, dynamical systems emerged as an independent mathematical discipline in the 1960s. In this exciting time of transition young mathematicians found this rapidly developing theory attractive and began to work in it under the direction of Sinai, Alexeyev, and others.

Katok received his doctoral degree from Moscow State University in 1968, and his dissertation reflected much-cited work with Stepin in *ergodic theory* about periodic approximations of probability-preserving transformations. The work has numerous applications and helped solve some long-standing problems that went back to von Neumann and Kolmogorov. It earned Katok and Stepin the prestigious prize of the Moscow Mathematical Society for young mathematicians, and Katok presented it in a 15-minute communication at the 1966 International Congress of Mathematicians in Moscow. He went well past the allotted time until he was stopped by a question: “Is this a short communication or an invited talk?”

At that time, Katok took a reading course on algebraic topology with Dmitri Anosov, a dynamicist at the Steklov Institute (the Mathematical Institute of the USSR Academy of Sciences), who played a central role in the creation of the modern theory of hyperbolic dynamical systems. Soon after, Katok joined Anosov in the invention of the Anosov–Katok “approximation by conjugation” method or “method of fast periodic approximations” for the construction of dynamical systems with interesting, often exotic properties. Katok wrote a complete 50-page draft of the paper in three successive evenings and later said that the “best way to overcome a depression is to sit and write a mathematical paper.” This work was called “the main event of the year in ergodic theory” and is an ingenious tour de force in *elliptic* dynamics. Meanwhile, Anosov’s influence and style drew

Katok’s interests toward *hyperbolic* dynamics, which made explosive progress in this period. The volume with Katok’s lectures from a 1971 summer school in Katsiveli (Ukraine) [17] became the main early Moscow text on differentiable dynamics. His lectures systematically present hyperbolic dynamics based on both some of the Western work and on the Anosov blueprint for studying the topological dynamics of hyperbolic sets using shadowing. This was much-needed and influential exposition but also cutting-edge mathematics, and notably, this summer school occurred a mere three years after Katok’s doctorate.

Searching the literature by authorship misses other essential contributions of his, however. As the mentor of Michael Brin and Yakov Pesin, Katok supported the development of the theories of both partially and nonuniformly hyperbolic dynamical systems. While in a (uniformly)

hyperbolic dynamical system each tangent space splits into complementary subspaces in which the action expands or contracts, respectively, a partially hyperbolic dynamical system includes a third “center” subspace in which contraction and expansion are allowed but at lesser rates than in the former subspaces. Nonuniformly hyperbolic dynamical systems are just that: they possess expanding and contracting subspaces as well, but there is no uniform control of them nor of their contraction and expansion rates.

While it is natural to expect that pushing the techniques from uniformly hyperbolic dynamics is the right strategy, the technical challenges are enormous. Pesin recalls struggling to explain his theory of nonuniform hyperbolicity in the publication, and Katok read drafts, discussed them generously, helped with the exposition and with putting this work in context—his vision and perspective were well ahead of Pesin’s at the time and crucial for this work. Yet, despite this deep involvement in the work, Katok refused to be named as a coauthor of what is now known as Pesin theory. This is why Brin and Pesin (and the Mathematics Genealogy Project) consider Katok their doctoral advisor even though he was not allowed to officially serve as such, and Anosov’s involvement was deep and crucial as well.

Not long after, Katok and Pesin discussed how to extend the theory of nonuniform hyperbolicity to systems with discontinuities. This had begun in the Anosov seminar and was motivated in great part by billiard systems. As they traveled to the seminar together one day, they used this lengthy commute to continue these discussions. Part

Katok with his children, their spouses, and his grandchildren near State College.

of this trip was a 30-minute bus ride, and it was on this bus that Pesin realized that the missing sufficient condition (beyond previously known assumptions on derivatives) was that the volume of a neighborhood of the discontinuity set be bounded by a power of its thickness. Just then, the passengers were asked to produce their tickets, and the pair realized that in their mathematical excitement they had forgotten to buy one. The fine was some 10 times as much as the bus fare, and Katok declared, "This is the price of a new mathematical discovery." Their convulsions of laughter over having been caught without tickets because of this breakthrough caused some puzzlement among the other passengers. Because the impending departure of the Katoks made it infeasible to collaborate on writing this result up together, they agreed that Katok would publish it himself abroad—a tremendous effort. The resulting book with Strelcyn has since been the standard reference on hyperbolic systems with singularities.

Katok was at Moscow University during the golden age of Moscow mathematics, and at times he explained that his choice of mathematics as a vocation was influenced by the relative freedom mathematicians enjoyed because their discipline was least affected and controlled by ideological impositions. He thought that otherwise he might have chosen to become a historian or a diplomat. To those who knew him, diplomacy seems a less natural fit, but he had an inclination to statesmanship when it was for a good cause. The Katoks visited Kiev in 2014, and then again in June 2015 after the Russian attack. A planned conference had been canceled, then postponed, because participation had collapsed in the face of news coverage. He encouraged others to go, spoke at the conference, and made a point of staying longer to show support for Ukraine and Ukrainian mathematics. He understood that sometimes just being there goes a long way. He was also an admirer and a real friend of Poland and of Polish mathematicians, and he had a passion for organizing doctoral schools and conferences in Będlewo near Poznań.

History remained a strong interest of his throughout his life, and stories abound about his prowess in that field. Many report on his command of the underlying facts, as demonstrated during a dinner conversation about nations that used to be part of the Soviet Union. When someone claimed that Armenia had "a population of 3 million," Katok interjected "Not 3 million, I don't think so, maybe 2.9 million at most," whereupon a quick internet search revealed that the most recently recorded population was 2.89 million. Another example involves a challenge to name all the US presidents, which Katok dismissed as trivial, proceeding to name them all, plus all vice presidents and all Roman emperors in order without hesitation. He apologized for not knowing in every case the exact years of their reign. In his first conversation with Katok, Mariusz Lemanczyk from Poland had to answer the "exam" ques-

tion of why the kingdom of Poland experienced exceptional prosperity during the second half of the 14th century (the answer being that the plague spared Poland). And when a conference in Będlewo featured a bus excursion, Katok took the microphone at the front of the bus to bring everybody up to speed on the relevant pieces of Polish history. "The drive was not short, and neither was the history lesson."

Of course, as in mathematics, recall of the facts is helpful, but understanding how to work with them is a higher skill—which Katok possessed in both mathematics and history. Around 1970 the well-known geometer Mikhail Postnikov gave a special lecture in Moscow on Morozov's "new chronology," a fringe theory in history, which had fallen into obscurity when Postnikov took it upon himself to bring mathematics to bear on this problem and rescue the theory. His lecture to this packed auditorium was compelling. After a few audience questions, Katok, a lowly junior mathematician and as such in the back of the auditorium, rose to say that he wanted to make some comments and that this might take some time. He then dismantled in a perfectly professional manner Postnikov's position on almost every single fact from the lecture. The audience was persuaded and looked at Postnikov for a reply. After a pause Postnikov said, "A week ago I gave the same presentation at a meeting of the Moscow Historical Society to professional historians. None of them presented such serious professional comments, and I have to think about them." Next, the well-known historian Lev Gumilyov came to the podium and said that he had been invited to present his view, but that his "young colleague" had already said everything he had to say.

Regrettably, mathematics was increasingly less sheltered by that time. From the late 1960s anti-Semitism and suppression of liberal thought grew at Moscow State University, and almost no Jews were accepted as students or faculty. So, Katok instead assumed an appointment at the Central Economics-Mathematics Institute (CEMI) of the USSR Academy of Science, which allowed him to combine work on mathematical problems in economics, if any, with research in pure mathematics. Katok had been attending the Alexeyev–Sinai seminar at Moscow State University regularly, and in 1969 he and Anosov started a seminar at the Steklov Institute (and moved to CEMI in 1975) with Katok as the driving force. At that time many mathematicians labored in day jobs unrelated to mathematics, and for them, these seminars were a vital connection to the research enterprise. In addition to talks about research by participants and guests, many seminar meetings were about works by other mathematicians, mostly foreigners, often based on preprints, which were rare and precious. The Alexeyev–Sinai and Anosov–Katok seminars were the main engine that put Moscow at the forefront of developing the modern theory of dynamical systems.

(Left) Katok in Leningrad, 1976; (Right) With Dmitry Anosov in Germany, circa 1981.

Katok's active seminar and conference participation is well remembered for the way in which it enriched the experience and value of a talk for both the audience and the speaker. He would comment on the meaning of a result, the context that makes it interesting, prior work on which it builds, and why it is important and interesting. Likewise I remember from his classes those distinctive moments when he turned from the board to face the audience and explain what it all means. He did not prepare the technical points of his classes in advance, and the proofs were presented off the cuff, which meant that details could be missing or incorrect. But proofs serve to illuminate, and their ideas as well as the rich context he provided gave a unique picture of the subject.

In my Caltech days Katok ran a seminar as well as a working seminar; in the latter one students learned the communal practice of mathematics, and they would often be organized around a topic. I also do not ever remember him taking notes in a seminar or any other context. He confessed to this as a weakness, but to my mind it always reflected two major characteristics. Thanks to his prodigious memory he did not need notes in the first place, and furthermore, his encyclopedic knowledge and vision of mathematics made everything new that he learned about an organic part of an interconnected landscape he consummately inhabited and in which he could place and locate each item at will.

Returning to the post-doctoral decade in Moscow, it is striking how Katok's work quickly spanned the breadth of dynamical systems well beyond his imprint on hyperbolic dynamics. He continued his work in *ergodic theory* with a substantial body of work on "monotone equiva-

lence" (often called Kakutani equivalence and based on a far-reaching generalization of the concept of time-change in flows), and his student Satayev did "excellent thesis work on Kakutani equivalence theory that came earlier than a similar project by the giants of ergodic theory D. Ornstein, D. Rudolph and B. Weiss and was only marginally weaker than theirs." A paper in *elliptic dynamics* astonishes experts to this day: He showed that without the required nondegeneracy assumption, the conclusions of the famed Kolmogorov–Arnold–Moser Theorem are completely reversed. His entry into parabolic dynamics proved lastingly influential as well: it involved the creation of the Katok–Zemlyakov "unfolding" method for the study of polygonal billiards; this idea was foundational for important parts of parabolic dynamics. Thus, within a decade from his doctorate, Katok was profoundly influential and internationally known not only in ergodic theory but in elliptic, parabolic, and hyperbolic dynamics.

Domestically, the situation started less ideally for the Katoks. In 1971 they had two children and lived in a single room of some 230 ft² in an apartment with two other families and a 4th room occupied until 1970 by Svetlana's grandparents, quite a step down from Anatole Katok's upbringing in a single-family apartment. Around that time, emigration became a realistic possibility, and some friends and colleagues started leaving. The Katoks considered the possibility but decided against it. Occasionally "cooperative apartments," whose construction began in the 1960s, became available for purchase, and the family was eligible for an apartment of some 800 ft². These were difficult to get, but they found a rather large and expensive one—for which

they saw no way of producing the required down payment of 40%, worth some 18 months of their gross household income. They recalled with gratitude that Anosov lent them the entire sum without interest and with an indefinite term of repayment.

Metropolitan Moves

In 1972 the Banach Center in Warsaw was established by the Academies of Sciences of Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Romania, and the USSR, with the aim of promoting international cooperation in mathematics, especially between the East and West. Anatole Katok's first trip abroad was for a 1975 conference there. By then his mother and brother had moved to the US, and the family's tolerance for life in the Soviet Union was wearing thin. They finally decided to leave the country, and when Katok returned to Warsaw in 1977 for an international conference with many participants from the West, he made no secret of the plans to emigrate. Several people brought this to the attention of the mathematics department chair at Maryland, William Kirwan (later provost, president, and chancellor), suggesting that Katok was a rising star with the potential to become a superstar and that he would have many offers from prestigious universities. Kirwan called Katok in Moscow. Katok had no idea who was calling nor how much in demand he was, so Kirwan's offer sounded very good to him. He accepted immediately.

The Katoks' last evening in Moscow, 1978.

On February 15, 1978 the Katoks emigrated from the Soviet Union. Vienna was the usual first stop on this exodus, and they still recall their joy at discovering a restaurant that offered a unique winning combination of cleanliness and price—and had the puzzlingly Scottish name of McDonald's! (Twenty years later, we made a pilgrimage together to this very McDonald's.) Next followed several weeks in Rome, and then a visit to IHES near Paris, where, for the first

Katok with his mother, Dora.

time in his life, Katok found himself with an appointment as a mathematician at a mathematics institute, and with an office of his own. Here he obtained some of his best-known and most widely quoted results on nonuniform hyperbolicity. Katok always recalled with gratitude the warmth with which the community of ergodic theorists in these cities received and supported them. That August, he turned 34, and the family arrived in Maryland, where Sasha and Dora, now residents of Rockville, picked them up at the airport. Anatole assumed his professorship, and Svetlana was finally able to pursue the doctorate in mathematics from which Soviet strictures had kept her.

A few remarks on some of the works from that period. Two *Annals* papers produced volume-preserving C^∞ Bernoulli diffeomorphisms on *any* compact connected C^∞ manifold, possibly with boundary.⁴ This is profoundly astonishing: these are volume-preserving diffeomorphisms that, up to a measurable change of "coordinates," are isomorphic to a Bernoulli shift, i.e., a much more complex counterpart to the probabilistic model of a fair coin toss. The first was written by Katok at IHES and did this on the disk and on surfaces and provided two steps of four in the later joint proof on any manifold [1]. Also partially written at IHES, Katok's by far most cited paper, "Lyapunov exponents, entropy and periodic orbits for diffeomorphisms," combined the shadowing approach of Anosov and Bowen (to whom the paper is dedicated—he died the year the Katoks arrived in the US) with Pesin theory to great effect. Its best-known results are Katok's Closing Lemma and that a diffeomorphism of a compact surface with positive topological entropy possesses a horseshoe. He was more proud of this work than anything else and lectured on it at the International Congress of Mathematicians in 1983 as well as the 1982 Rufus Bowen Memorial Lectures at Berkeley.

⁴Except in dimensions 3 and 4, where Dolgopyat and Pesin established this and more [6].

A 1982 paper gave effective lower bounds on the growth of the number of closed geodesics on a surface and proved that for a higher-genus Riemannian surface the topological and Liouville entropies agree only if the curvature is constant.⁵ With the help of Ralf Spatzier, Katok computed both entropies for all locally symmetric spaces and found that they agree. The resulting table is followed by the comment

It looks like a reasonable conjecture that those are the only cases of manifolds of negative curvature for which the Liouville measure has maximal entropy.

Before long, this understated aside came to be known as the Katok Entropy-Rigidity Conjecture. Rigidity theory has been active ever since, thanks on no small part to Katok himself. He not only worked on this conjecture but put much effort into popularizing it by giving talks, organizing or participating in a series of conferences over two decades, and directing the interest of mathematicians towards the conjecture. Rigidity theory was becoming an industry, and has been going strong since.

True to form, simultaneously with these papers on hyperbolic dynamics, Katok did notable work on both elliptic and parabolic dynamics as well as ergodic theory; the latter, in the spirit of his quest to understand which phenomena from ergodic theory are realized in smooth dynamics, shows that there are diffeomorphisms with the Kolmogorov property that do not have the aforementioned Bernoulli property.⁶

Academic appointments in the US offered entirely new outlets for Katok's legendary organizational and collaborative energies. While he always maintained intense seminar activities, he immediately set about organizing special years such as 1979–1980 at Maryland and 1983–1984 at the Mathematical Sciences Research Institute.⁷ He was able to travel extensively, and rare was the year in which he did not visit multiple continents. Visits to Warwick in 1979 and 1980 led to the founding of the *Journal Ergodic Theory and Dynamical Systems*, which for some four decades now has been the global journal of record for the field. Twenty-five years after this founding, he did it again and started the *Journal of Modern Dynamics* with a view to being a highly selective dynamical systems journal with a broad focus and a democratic editorial board.

In the US, Katok was also in a position to build a research group. Maryland made significant hires during his time there (including Brin), and he began to have postdocs in

Jacob Feldman, Anatole Katok, and Donald Ornstein planning the 1983–4 special year at MSRI during the conference in honor of S. Kakutani at Yale University, 1982.

addition to graduate students. Like others since, they recall that he always made time for talking about mathematics or career advice, even when he was obviously busy running a conference or in the midst of a collaboration with someone else. He would come in early or stay late, or find time during an excursion, whatever was needed. And he had a knack for cultivating talent and adjusting his mentoring to the students and their abilities.

Svetlana obtained her doctorate from the University of Maryland in 1983 (with Don Zagier), whereupon the Katoks departed for a year in Berkeley. Anatole was at MSRI, while Svetlana was a lecturer at UC Berkeley. For many years thereafter, Katok maintained close relations with MSRI, including service as a trustee. His time there during the 1992 special year on *Lie Groups and Ergodic Theory with Applications to Number Theory and Geometry* is worth mentioning for several reasons. One of these is that his facility with ideas is illustrated by an episode that started here and then in response to a question put to him by Keith Burns and Livio Flaminio. Without hesitation, he produced an example of a continuous foliation with smooth leaves and a set of full Lebesgue measure that intersects each leaf in a single point(!). Flaminio and Burns wrote this up, titled it “Fubini’s nightmare,” and did nothing with it until 1996 when Burns sent it to his colleague Amie Wilkinson, and via Charles Pugh and Michael Shub the example made its way to John Milnor—who immediately wrote it up for the *Mathematical Intelligencer* [31] under the alliterative title “Fubini foiled.” This “pathological” phenomenon became instantly famous and has since been discovered to be somewhat typical of the center foliation (if defined) of a partially hyperbolic dynamical system.

In 1983–84 the need of the Katoks for two jobs in one region was no secret, and Barry Simon initiated an offer of

⁵The acknowledgments show that the ability to travel internationally and invite visitors to Maryland from the world over made a real difference in what Katok was now able to do.

⁶These are very close notions, and even in a purely measure-theoretic context it is not a simple exercise to find such an example.

⁷www.msri.org/programs/100.

a professorship for Anatole at Caltech, where they moved in 1984, Svetlana having a two-year adjunct position at UCLA. Now 40, Anatole settled into this position, while over time Svetlana was to hold faculty positions at most University of California campuses. From this time onward, Katok maintained a continuous stream, if not torrent, of doctoral students, with a grand total north of 40. For most of us this afforded a communal experience not entirely unlike what the Moscow seminars must have been like. This was not only due to the number of doctoral students but also to his ability to build a research group by hiring postdocs and faculty, inviting visitors, and drawing on interactions with related disciplines. A few salient items from my career may give an indication. Having come to the University of Maryland as a physics student on a Fulbright scholarship, I happened to hear that “someone called Katok” was going to give a course on classical mechanics in the spring of 1983. While taking it, I decided to become a mathematics doctoral student at Maryland, and in the spring of 1984 I got a letter from MSRI. Katok wrote that he was moving to Caltech and invited me to join him there. When I arrived, a former student of Simon’s began to work with Katok as well. Soon after, postdocs and faculty joined, and the number of doctoral students rose quickly. There was a steady stream of high-caliber seminar speakers and medium-term visitors from around the globe plus an active working seminar and almost annual courses on dynamics by Katok, never twice the same. He naturally interacted beyond dynamical systems. Simon was his office neighbor, and their animated conversations often rang through the hallways, the pitch ever increasing with the excitement of the interaction.

From the Caltech days, a few larger mathematics projects are worth singling out (in addition to the completion of [30]). With Gerhard Knieper, Mark Pollicott, and Howard Weiss, Katok carefully studied the dependence of entropy on perturbations of Anosov flows, and with Hurder he deeply connected dynamics with foliation theory [13, 14]. Their study of the Godbillon–Vey class became seminal

in an unintended direction: it necessitated a more careful study of the (transverse) regularity of the invariant foliations of an Anosov 3-flow, and in so doing led to another rigidity conjecture: that an Anosov flow is smoothly conjugate to an algebraic one if these invariant foliations are transversely C^2 . Kanai almost immediately made a key construction (the Bott–Kanai connection) that enabled him to prove the first such result for geodesic flows on manifolds of dimension greater than 2. He came to Caltech as a Bateman Instructor, and Katok and Feres embarked on the chase of the comprehensive such result. And a chase

With Werner Ballmann in Oberwolfach, 1987.

it was: they were aware that Foulon and Labourie, soon joined by Benoist, were onto the same target. The Feres–Katok strategy was to buttress Kanai’s line of argument with ever more sophisticated dynamical refinements to progressively weaken the needed curvature-pinching hypothesis. Some of this work took place during a summer in Göttingen, and I remember the exhilaration of progress as well as the competitive spirit. In the end, Benoist–Foulon–Labourie got there first, and some of the Feres–Katok work remained unpublished. The consolation was that the French team had gotten there using a “big hammer,” the open-dense orbit theorem of Gromov. But they had also far exceeded the target by treating contact Anosov flows, i.e., they obtained rigidity without assuming the structure of a geodesic flow in the first place.⁸

The Göttingen memories also bring to mind the Oberwolfach conferences “Dynamische Systeme” then organized by Moser and Zehnder—I remember a reckoning that revealed Katok to have been second only to Zehnder in faithfulness of attendance. Helmut Hofer, who now co-organizes these, likes telling the emblematic story of the “Katok diagonal argument.” Katok’s usual seat was in the front right (sitting in front was his general custom, and the doors being on the right in the Oberwolfach meeting room makes this the natural side for late-comers). Michael

⁸Contact Anosov flows whose invariant subbundles are C^∞ or highly smooth are (essentially) smoothly conjugate to the geodesic of a locally symmetric space. Whether C^2 suffices remained open.

Herman, another prominent regular, might sit in the very left rear corner. It was not uncommon for one of Katok's amendments to someone else's talk to be countered by Herman from the opposite corner: "I can't believe that I am hearing this! I have three counterexamples to that statement. No, I have infinitely many counterexamples to what you are saying...", and so the argument across the diagonal of the room was on. This is memorable, of course, not only because it is amusing but because these interactions were illuminating for all those present and emblematic of the creative spark of those meetings.

Helmut Hofer telling of the "Katok Diagonal Argument."

Middle Pennsylvania

"Moscow, Vienna, Rome, Paris, Washington, Los Angeles—*State College*???" was a refrain briefly heard among dynamacists in 1990. The intimacy of Caltech's mathematics department made it challenging to build and maintain a large research group, and Svetlana's position at UC Santa Cruz made for a longer commute than a family with a young child would want to keep up. Richard Herman from the Pennsylvania State University engineered a double-offer that drew the Katoks to State College, but their previous metropolitan biography raised doubts about how long they would last there. Though they spent nary a summer there, they never left. And they fully engaged from the start, helped by the simultaneous arrival of Pesin from Moscow via Chicago and H. Weiss from Caltech. They went on to build the department into the most prominent center of dynamical systems in the country. Mathematically as well, one might call this a new period.

In addition to the aforementioned seminal rigidity work and a continuing stream of publications in various parts of dynamical systems, Katok became engaged with the Zimmer program, the core conjectures of which were only just yielding at the end of his lifetime [2,3]. He heartily jumped in together with others by dedicating the first Dynamical Systems seminar to the "super-mathematics" of the day, Margulis–Zimmer superrigidity, following Robert Zimmer's

book. This continued for two years and then slowly morphed into a more general Dynamical Systems seminar with rigidity a cornerstone for years to come. The first meeting was packed with about 20 people, quite unusual for a topic of such depth, which required expertise in ergodic theory, functional analysis, Lie groups and algebras and their representations, unitary representations of groups, algebraic groups, and algebraic geometry. Katok kept the participants together, and encouraged everyone to participate in order to build a research school, even though he did not expect all the participants to write papers in the field. Remarkably, over the years most of the participants actually published in rigidity theory, and many defended PhD theses under his supervision.

Indeed, probably Katok's most impressive contribution to dynamics in terms of research papers is the program on rigidity of actions of higher-rank abelian groups, i.e., to show that faithful actions of $\mathbb{R}^k$ or $\mathbb{Z}^k$ are standard. He carried this out in large part with former students, e.g., in more than twenty papers with Spatzier, Damjanović, Kalinin, Niţică, Török, and Kononenko. He also hired and collaborated prolifically with Manfred Einsiedler and Federico Rodriguez Hertz, mostly on rigidity questions, and Rodriguez Hertz has since succeeded him on the Raymond N. Shibley Chair. Compared to higher-rank lattice actions, he saw abelian actions as a more important (broader) subject because he wanted to understand dynamics from many different sources, and in abelian actions all types of dynamics appear and can be understood. It was icing on the cake rather than justification, that this could be applied to number theory when he, Einsiedler, and Elon Lindenstrauss addressed the Littlewood Conjecture. (Lindenstrauss went on to win a Fields Medal.) Icing as well, that this turned out to contribute to the recent breakthrough on the Zimmer program. Interestingly, Aaron Brown, who was central to proving the Zimmer conjecture, told me that all the tools he needed, he learned from Katok. Among these many works, the work with Spatzier on measure rigidity [28] and cocycle rigidity [29] started these developments, and his work with Kalinin and Rodriguez Hertz on nonuniform measure rigidity [15] then opened the door to the Littlewood conjecture and to the Zimmer program.

Throughout, Katok vigorously recruited students, and the vast majority of his doctoral students graduated from the Pennsylvania State University. Among Alena Erchenko's early memories is that Katok told her at the beginning of the PhD program: "You will never know as much as I do. Therefore, the best route is to learn something that I do not know." She and I think he liked when his students were able to prove something that he thought should be different. In her work with him, it first seemed that there might be restrictions on the pairs of Liouville and topological entropies, but then they discovered models that allowed them to vary Liouville entropy without changing topological

entropy much. Likewise, my dissertation result involved a geometric “threading” mechanism rather than one of a cohomological nature as in any previous work. Another memorable piece of advice was given at home, when he entertained Jana Rodriguez Hertz and Raúl Ures: “If you want to succeed in mathematics, then one possibility is to be Margulis. But if, like in my case, you are not Margulis, then you have to work like crazy. There is no other way.”

His care for others in the community went well beyond mathematical needs. H. Weiss called Katok the most gregarious mathematician he has known, and the frequent social events hosted by the Katoks were an important center of a community. He helped his students with moves and medical appointments, and he made a point to include young mathematicians in conferences, over time he wrote letters of recommendation for most dynamacists, and at times his active intervention rescued careers.

My largest project with Katok mainly took place during the Penn State years. In our Caltech days we had vaguely entertained the idea that my polished notes from his first dynamical systems course there might become a book, but we did not make concrete plans until 1989. Helped by notes from his dynamics courses since, we had a strong base for modest plans. It surely helped that we had no idea what we were getting ourselves into. Mostly in the course of mutual visits over five years, *Introduction to the Modern Theory of Dynamical Systems* [24] acquired its final form and 800-page heft. By now, it is The Book on dynamics, and most of those who earned PhDs in dynamical systems in the present millennium grew up in no small part with this book. On a 1996 visit in Montevideo we dug into a follow-on project, *A first course in dynamics | with a panorama of recent developments* [12], which had half the size—but took twice as long. Other book(-size) projects of his include the 200-page *Principal Structures* [11], which provides a common background for two Handbook volumes [8,9], the 130+ page current-research survey *The theory of dynamical systems and general transformation groups with invariant measure* [27] from Moscow days, and the monographs *Invariant Manifolds, Entropy and Billiards. Smooth Maps with Singularities* [30], *Rigidity in higher rank abelian group actions. Volume I. Introduction and cocycle problem* [26], and *Combinatorial constructions in ergodic theory and dynamics* [19]. Two more books are worth mentioning in the context of the Katoks’ imprint on undergraduate mathematics education at the Pennsylvania State University. They created the Mathematics Advances Study Semesters program (MASS), which provides a springboard to graduate education for many students from small colleges who might otherwise not be prepared or inspired for graduate education, and two courses Katok taught in it resulted in books [4,23] with Vaughn Climenhaga, then a graduate teaching assistant for the course.

Katok in his office, 2002.

At the Pennsylvania State University Katok was able to build lasting structures to support dynamical systems worldwide. One is the aforementioned Center for Dynamical Systems and Geometry. The other is a major annual workshop. Upon arrival at the Pennsylvania State University, Katok set upon organizing a large rigidity conference for March 1991, and it was promptly followed by a Penn State–University of Maryland Workshop in Dynamical Systems and Related Topics in October 1991, which has taken place annually ever since and is always followed by a companion meeting at the University of Maryland in the Spring. Participants travel from one of these institutions to the other, and they are joined by participants from across the US and the globe. This alone puts Penn State on the mental map of every dynamacist in the world. To raise the visibility of dynamical systems, Katok and Pesin persuaded Brin to create an international prize for outstanding work in the theory of dynamical systems and related areas. The first Michael Brin Prize was presented in 2008 at the Maryland session of the semi-annual workshop, which was dedicated to Brin’s 60th birthday, and this (now annual) prize has since been complemented by the annual Michael Brin Dynamical Systems Prize for Young Mathematicians.

Together with the countless doctoral students and postdocs who cycled through the department in these

At the Pennsylvania State University during the 2015 Fall Workshop with Michael Shub and Yakov Pesin.

three decades, plus those hired during his time, whether still at Penn State or not, Katok made Penn State a center of dynamics, and dynamics central to Penn State—with a broad perspective. In the estimation of George Andrews, a former department chair, “The rise of the reputation of our department owes much to his leadership both in his own research and teaching and in bringing outstanding mathematicians to our department.”

He had a zest for life that had many dimensions. He loved conversations (about mathematics, history, and almost any other topic), art, music, wine, books, and food—locally, abroad, and at home. The aforementioned Jana Rodriguez Hertz, an Argentinian mathematician from Uruguay, recalls dinner at his house: “He cooked steaks, and he really knew how to do it. And let me tell you that it is not easy to impress an Argentinian or Uruguayan at that.”

While an inveterate urban traveler, Anatole also enjoyed the great outdoors. Many remember the obligatory vigorous conference hikes, such as the clammers up the San Gabriel Mountains or the workshop Sundays in the hills of Pennsylvania. Alas, in the mid 1980s this had to pause when Katok underwent aggressive cancer treatment. Yet, he came to every seminar and continued to work as usual, so the gallows humor in the research group had it that this crisis had reduced Katok’s energies to those of a mere earthly being. He was before long pronounced fully cured, but resulting heart and lung problems started appearing a decade later and increased over time. Nevertheless, he persistently threw himself into life and mathematics. Aside from our second book, we completed several other large projects and started yet others now left in limbo. All this alongside his extensive research productivity on higher-rank rigidity and many other topics, plus an unabated flow of books and expository writing, including historic/sociological works with unique and thoughtful perspectives that will likely remain a useful part of our historical record [10, 16, 20–22, 25]. On my last visit we reworked a perennial

book project into a different form that would make it more feasible in reasonable time, but he warned me that because he was doing well enough to work on such projects, he was going to have a few others on his front burner. More recently, the Swedish Research Council appointed him to the Tage Erlander Guest Professorship 2018 in recognition of his excellent services to science, and he was scheduled to consummate in the second half of the calendar year. His enthusiasm and engagement never stopped, but this was not to be, and numerous projects are now suspended.

Anatole Katok is gone, but he created enough momentum, infrastructure, and leadership talent that we can expect dynamical systems to do him proud in years to come.

NOTE. An expanded version of the article will appear in *Ergodic Theory and Dynamical Systems*.

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Credits

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