Dance Your PhD: Representations of the Braid Groups

Nancy Scherich

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EDITOR'S NOTE. See the interview of Scherich by Rachel Crowell at www.ams.org/news?news_id=3826.

I learned about the Dance Your PhD competition many years ago, but thought it would be too impossible to turn math research into dance. This past June, my boyfriend, Dean, forwarded *Science* magazine's announcement of their 2017 competition to me and encouraged me to make a submission. He said if ever there was someone to figure out how to blend math and dance together, it would be me. I have been a dancer and performer all my life, and about a year and a half ago I started aerial dance lessons. Dean suggested that I use aerial dance to describe my work with braids.

At first I was reluctant. But the more I relaxed and gave myself permission to think outside of my math box, the more the ideas came to me. Sometimes I think all you need is the right encouragement at the right time. In my video,¹ I chose to describe braid group representations and focus on the property of faithfulness, as my research is highly motivated by the infamous open question of faithfulness for the Burau representation of braid groups for $n = 4$.

Much like the braids in one's hair, a braid is a diagram of tangled strands. The braid group on n -strands, denoted B_n , is a group whose elements are certain equivalence classes of all the braids made with n strands. The group operation is vertical stacking of braids. Alternatively, the braid group can be presented by $n - 1$ generators σ_i and

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¹www.sciencemag.org/news/2017/11/announcing-winner-year-s-dance-your-phd-contest

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Figure 1. Scherich was the overall winner across all disciplines in the 2017 *Science* magazine Dance Your PhD competition with her creation of and dancing in her video “Representations of the Braid Groups.” The announcement says, “It involves linear algebra and murder.” According to the video, many kernels are annihilated, but “some are extremely clever and hide in the matrices. It remains an open question for mathematicians to find these kernels.” The credits include her advisor Darren Long and her mom.

relations:

$$\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \quad \text{and} \quad \sigma_i \sigma_j = \sigma_j \sigma_i$$

for i in $\{1, 2, \dots, n - 2\}$ and $|i - j| \geq 2$, as in Figure 2.

A common way to study the braid groups is to look at their representation theory. The Jones representations of the braid groups are the representations where the generators have two eigenvalues. It turns out that much is known about these representations, and in fact, they are parameterized by the Young tableaux in the same way that the Young tableaux parameterize the representations of the symmetric group. The Burau representation is one of the Jones representations. All of the Jones representations have a variable called q . The topic of my thesis is to find

Figure 2. The braid group can be described by the two pictured types of relations: $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$ and $\sigma_i \sigma_j = \sigma_j \sigma_i$.

careful specializations of q to certain algebraic numbers which then force the representation to map into a lattice.

Image Credits

Figures 1 and 2 courtesy of Nancy Scherich.

Photo of Nancy Scherich courtesy of Dean Morales.

ABOUT THE AUTHOR

Nancy Scherich's interests are low dimensional topology, planar algebras, quantum computation, dancing, aerial acrobatics, sewing, and welding.

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