Mathematical and Computational Modeling of Microorganism Swimming

Ricardo Cortez

Equations that describe the motion of fluids in three-dimensional spaces, much like the planar flow around the helical swimmer of Figure 1, are based on Newton's second law where the forces include those from pressure gradients, friction due to viscosity, inertia, and external forces exerted on the fluid. Depending on the application, some of these

Figure 1. A computational helical swimmer and fluid flow pattern (velocity field and streamlines) on a plane.

forces may be negligibly small. On tiny scales, as in the case of biological flows around microorganisms, the inertial forces can often be omitted leading to the Stokes equations for incompressible flows:

$$0 = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{F}, \quad 0 = \nabla \cdot \mathbf{u},$$

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where p is the pressure in the fluid, μ is the viscosity, $\mathbf{u}$ is the fluid velocity, and $\mathbf{F}$ is an external force density, which can be time-dependent and drives the flow.

Since the fluid velocity depends linearly on the external force, numerical methods for computing the solution of the Stokes equations can be designed based on their fundamental solution. At the Fall Southeastern Sectional meeting (November 12–13, NCSU), I will focus on the development and application of the method of regularized Stokeslets, which is based on a systematic derivation of nonsingular versions of the fundamental solution of the Stokes equations. Given a smooth function ϕ_ϵ (e.g., a Gaussian) that approximates a delta function as a distribution, we derive the exact solution $(p, \mathbf{u})$ of the Stokes equations for the forcing $\mathbf{F}(\mathbf{x}) = \mathbf{f}\phi_\epsilon(\mathbf{x} - \mathbf{x}_0)$. The resulting formula for $\mathbf{u}$, called a regularized Stokeslet, has no singularities and thus can be used to compute flows where the force density $\mathbf{F}$ is distributed over surfaces, curves, or even scattered points.

The regularization parameter ϵ represents roughly the radius of a ball where a force is spread so that $\epsilon \rightarrow 0$ reduces $\mathbf{u}$ to the (singular) fundamental solution. In studying the flow generated by beating or pulsating flagella we use this notion to set the value of ϵ based on physical parameters. My presentation will include examples of the usefulness of the method of regularized Stokeslets (and some variants) in biological applications.

Credits

Figure 1 reprinted from *J. Comput. Phys.*, v.317, J. K. Wróbel, R. Cortez, D. Varela, L. Fauci, Regularized image system for Stokes flow outside a solid sphere, pp. 165–184, (2016), with permission from Elsevier.

Photo of Ricardo Cortez and student, courtesy of Omid Forouzan.

Ricardo Cortez

with his student Ellie Ahmadi, who finished her PhD in 2016.

ABOUT THE AUTHOR

Ricardo Cortez is an applied mathematician working on the development and analysis of numerical methods for the computation of fluid motion with applications to biology. Here he is shown

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