Elisenda Grigsby Interview

Photo courtesy Lee Pellegrini.

Elisenda Grigsby is associate professor at Boston College. She won an NSF CAREER grant in 2012, and more recently the inaugural AWM-Joan and Joseph Birman Research Prize.

Diaz-Lopez: When did you know you wanted to be a mathematician?

Grigsby: Not until pretty late. I always knew I liked science, but when I first got to college I planned to major in biochemistry. I ended up taking this amazing seminar-style course led by David Layzer (an astrophysicist) and Cynthia Friend (a chemist), consisting of twice-weekly meetings to discuss together how we'd tackle some really hard (but fun!) problems.

After that course, I decided to major in physics and ended up taking a number of math courses to go along with it. But I burned out halfway through my junior year and took a semester off from school to do something completely different. I got a plane ticket to London and a six-month work permit. I spent the semester waitressing, and ended up reading topology textbooks in my spare time.

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When I came back, I decided to switch my major to math. At that point, though, I was completely outside of the math community at my school—a lot of the majors had gone through the standard math sequence during their first year, and I didn't know any of those people, so I felt a bit isolated. By the time I graduated, I had decided not to pursue a PhD. I liked math, but I still had the impression that I wasn't good enough at it to actually do it for a living.

Instead I worked for a year doing decision analysis/operations research at a company in Silicon Valley. This was a great experience, but the main thing it taught me was how much more fun I was having back when I was doing math. A public lecture by Brian Greene that I attended that fall at Stanford clinched it. I applied for grad school that winter, and felt very lucky to be accepted by Berkeley—which ended up being the perfect place for me.

Diaz-Lopez: Who encouraged or inspired you?

Grigsby: So many people that I can't possibly list them all without accidentally leaving off someone important, so I'll just list the obvious ones. David Layzer was the first: his Chem 8/9 course was really a turning point for me, and he went out of his way to convince me that I belonged, at a time when I really didn't think I did. Then my undergraduate thesis advisor, Ken Fan: he knew how to encourage and direct me without being overbearing about it. As a graduate student: of course my two advisors Rob Kirby and Peter Ozsváth. Their styles of doing mathematics are so different from each other, but it was a pleasure to watch both of them at work and to get to talk to them so often.

I think this is the wonderful thing about the mathematical community: once you have become interested enough in a subject that you are able to start making your own dent in it, most people are excited to talk to you and to help you succeed. Of course, you have to show that you're willing to work and think on your own. But the more you put in, the more you get out.

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Diaz-Lopez: How would you describe your research to a graduate student?

Grigsby: I study objects in low-dimensional topology (e.g., 3-manifolds, 4-manifolds, links, and braids) using “homology-type” invariants. Briefly, one associates an abstractly-defined chain complex to some collection of data describing a topological object (e.g., a Heegaard diagram for a 3-manifold, a diagram of a link). The homology of this chain complex ends up being an invariant of the topological object (i.e., does not depend on the choices involved in its definition). These homology-type invariants are inspired by ideas in physics—in particular, quantum field theory and gauge theory.

Diaz-Lopez: What theorem are you most proud of and what was the most important idea that led to this breakthrough?

Grigsby: I’m partial towards some work I did recently with Tony Licata and Stephan Wehrli, but I think one is always biased towards recent work. This is what I’m thinking about these days, because this is what I like best!

We proved that a particular homology-type invariant (sutured Khovanov homology) associated to an n -cable of a knot (an n -cable of a knot is, roughly-speaking, n parallel copies of the knot, see Figure 1) admits commuting actions of the Lie algebra sl_2 and the symmetric group S_n . In the case of the trivial knot (aka the “unknot”), these actions agree with the classical commuting actions of sl_2 and S_n on the n -th tensor power of the defining representation of sl_2 : one instance of a so-called “Schur-Weyl representation.” So one can think of this construction as giving “knotted” Schur-Weyl representations.

As for the breakthrough(s) that led to the work: Stephan and I had been studying this particular homology-type invariant for a while (ever since 2008), and we got to talk to Tony about it while we were at MSRI [Mathematical

Image courtesy of E. Grigsby.

Figure 2. Identifying the boundary arcs of this octagon gives a genus 2 surface. The curves indicate how to attach handlebodies to form a 3-manifold, and the starred points specify a knot in the 3-manifold. Ozsváth and Szabó showed how to use such data to define a homology-type invariant of the knot. This particular manifold has a Z_2 symmetry covering the 3-sphere with fixed point set the knot.

Sciences Research Institute in Berkeley] in 2010. Tony is a geometric representation theorist, so he had a very different perspective on things, and Stephan and I both found it useful and interesting to chat with him periodically, but nothing happened at the time. In the summer of 2012, he told us that there ought to be an action of sl_2 on the homology-type invariants we were studying. The construction was pretty abstract, and there were details we weren’t quite sure how to fill in, so we didn’t know what to do with it. But the idea that there could be such an action was really intriguing.

Then one morning (probably about 2 months later!) right after I woke up, I realized that there was a completely obvious, completely concrete, sl_2 action right at the chain complex level that was staring us in the face the whole time. It also turned out that Stephan had worked out the details of an S_n action on the homology of n -cables way back in 2005. With a little further thought, it was clear that his action commuted with the sl_2 action. Once we started realizing things, everything came together in just a few weeks.

But (as is the case with most results), that few weeks was several years in the making! You’re spinning your wheels most of the time, thinking you’re not moving. But secretly you are.

Diaz-Lopez: What advice do you have for graduate students?

Figure 1. A 3-cable of the right-handed trefoil knot. Licata, Wehrli, and Grigsby recently found commuting actions of the Lie algebra $sl(2)$ and the symmetric group S_n on a variant of the Khovanov homology of the n -cable of any knot.

Image from "Visualization of the Genus of Knots," by Jarke J. van Wijk and Arjeh M. Cohen, in *Proceedings IEEE Visualization 2005*, edited by C. Silva, E. Gröller, H. Rushmeier, IEEE CS Press.

Figure 3. A smoothly imbedded, oriented surface in the 3-sphere bounded by the $(7, 2)$ torus knot. A conjecture of Milnor proved by Kronheimer-Mrowka states that this surface has the smallest possible genus among smoothly imbedded surfaces in the 4-ball with this knot as boundary.

Grigsby: Talk to as many people as you can. That's the only way to find out not only what the community finds interesting but also what you find interesting. And if you don't find a problem interesting, it's really hard to expend the energy required to solve it. Having a good "nose" for problems and the tenacity to keep chewing on them—within reason—is a much more important trait than being quick. Of course, quickness helps, but depth is better.

Diaz-Lopez: All mathematicians feel discouraged occasionally. How do you deal with discouragement?

Grigsby: Not very well. But after you've been around for a while, you eventually realize that it's all part of the process. As mathematicians, it nags at us if we don't understand something as well as we'd like, and I've often found myself unable to get up from my desk when I'm

stuck. But usually the right thing to do when you're stuck is to step as far away from your desk as possible. Otherwise you'll never get out of the particular rabbit hole you're in. Breakthroughs often come in the first five minutes after taking a break.

Also, Paul Melvin once told me (and I think this is right on): Periodically remind yourself how much more you know today than you did a year ago today. That will raise your spirits.

Diaz-Lopez: You have won several honors and awards. Which one has been the most meaningful and why?

Practically speaking, the CAREER [from the National Science Foundation]. Who knows if I'd have tenure now if I hadn't gotten that. Putting together the proposal also gave me a chance to reflect on my goals, research-wise and otherwise.

The Birman Prize [from the Association for Women in Mathematics] was also very personally meaningful, mostly because I have the deepest respect for Joan Birman—both her mathematics and her life story are amazing. I think the interview she gave in the *Notices* of the AMS back in 2006 should be required reading for all graduate students. She's a giant in the field, and she didn't even begin her PhD studies until after the age of thirty.

Diaz-Lopez: If you were not a mathematician, what would you be?

Grigsby: If I were eighteen again, I'd probably want to get an engineering degree, working on developing alternative energy sources. Either that or machine learning and A.I.

Diaz-Lopez: If you could recommend one lecture to graduate students, what would it be?

Grigsby: I'm topologically-biased. I loved Bott and Tu's *Differential Forms in Algebraic Topology* back when I was a grad student. Also anything by Milnor: e.g., *Topology from the Differentiable Viewpoint*, *Morse Theory*, and *Singular Points of Complex Hypersurfaces*.

Alexander Diaz-Lopez is a PhD student at the University of Notre Dame. Diaz-Lopez is the first graduate student member of the *Notices* Editorial Board.