How Not to Be Wrong

Reviewed by Andrew J. Blumberg

How Not to Be Wrong: The Power of Mathematical Thinking

Jordan Ellenberg

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We are witnessing an interesting historical moment concerning public engagement with mathematics. Not only is daily life permeated by the applications of sophisticated mathematics, but mathematics is increasingly part of our cultural conversation and present in the public sphere. And yet we all know that for too many of our students, we are failing to communicate both the beauty of mathematics and real competency in mathematical reasoning. In particular, the sense of play, exploration, and discovery that is the central experience of the research mathematician is rarely conveyed effectively. It's a frustrating situation: our students are immersed in mathematics, but few appreciate what it is or how they could engage with it. I'm reminded of David Foster Wallace's version of a very old joke, in which an old fish asks some young fish, "How's the water?," to the baffled response of "What's water?"

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It is explicitly against the backdrop of this paradoxical state of affairs that Jordan Ellenberg has written his wonderful book *How Not to Be Wrong*. As the title suggests, the book takes as one point of departure a pragmatic view of mathematics as a conceptual framework for organizing and trying to understand the wildly confusing and incomplete data provided by the world. But Ellenberg's text is a multifarious enterprise: it simultaneously advocates for the value of research mathematics, explains what mathematics can do, encourages readers to experiment and view mathematics as something that they too can make their own, and strives to reveal the surprising beauty of mathematical explanation. Remarkably, the book succeeds on all of these fronts.

In the service of "not being wrong", the book emphasizes a basic message: when trying to understand the world, it's easy to fool yourself and very tempting to do so when you've got something at stake. As one would expect,

The Fano plane.

there is a heavy emphasis on probability and statistics: entertaining discussions of regression and linearity fallacies, the problems with statistical tests and p-values, correlation vs. causation, the virtues of Bayesian inference, paradoxes in voting and aggregating public opinion, lotteries

and expected value, and so forth. In and of themselves, these are just the sorts of topics one might expect in a book about the use of mathematics aimed at a broad audience. For that matter, many of them parallel a good introductory statistics class. But Ellenberg weaves these together very skillfully and effectively conveys the central message that mathematical tools allow us to answer questions but also have serious limitations. Moreover, a big part of the pleasure of Ellenberg's work comes from the very interesting examples he introduces in the service of his narrative. Some of these are standard: for instance, the stock advice scam involving sending predictions first to 1,000 people, then to the roughly 500 who received correct predictions, until finally you sell your advice to the lucky "survivors". But many of them are more novel, ranging from the entertainingly idiosyncratic (e.g., look for the discussion of the Cat in the Hat when he talks about the risks of bad priors in Bayesian inference) to the remarkably deep.

For an example of the latter, the most surprising and beautiful part of the book is the discussion of the link between error-correcting codes, the lottery, and projective geometry. Since this is such a fantastic example, let me try to convey its flavor. Imagine a lottery game in which the state picks n numbers from the range $[0, m]$ (for $m > n$), and a player buys a ticket with n numbers (of their choosing) on it. The player wins if at least k of the numbers on the ticket match the state's n winning numbers. The question now is the following: if you were going to buy a large number of tickets, what's a good strategy? One possibility is to buy tickets randomly. But it would be better to try to arrange ticket purchases such that all the winning subsets of numbers are covered and appear the same number of times; this minimizes variance in the return.

Trying to generate such a set of tickets by exhaustive search is infeasible due to combinatorial explosion. But Ellenberg turns to a discussion of the projective plane and then finite geometries: finite sets with specified points and lines such that each pair of points lies on a unique line and each pair of lines intersects in a unique point. Using the example of the Fano plane (which has seven points and seven lines, each of which has three points), he observes that in the case when $m = 7$, $n = 3$, and $k = 2$, choosing a ticket for each line results in a set of tickets such that each pair of numbers occurs exactly once. Of course, the Fano plane is too small to be useful; the question of how to find large finite geometries now arises.

Ellenberg then moves on to connect this problem to coding theory; a solution is provided by choosing each ticket to be a codeword in a good error-correcting code (one where the codewords are well separated in the Hamming distance). After explaining coding and something about information theory, he finally also connects this back to geometry via sphere-packing problems and recent progress on optimal packings.

Even when the examples are standard, the explanation is often surprisingly thoughtful. For instance, the most interesting aspect of Ellenberg's discussion of Galton's discovery of the phenomenon of "regression to the mean" is his explanation of the elliptical boundary region for bivariate normal variables. He observes that the ubiquity in applications of simple mathematical objects (e.g., conic sections) arises from the fact that there are comparatively few simple mathematical objects—once you know something has such a description, there aren't so many options!

These discussions give a flavor of the interconnectedness of mathematical ideas. They also communicate the magical nature of understanding. And, indeed, throughout the text there is emphasis on the sheer joy of mathematical discovery and the ease with which one can learn to do this in small examples. As an aspect of this, Ellenberg works to make it clear to ordinary readers why the pursuit of mathematics is important and why even seemingly "nonapplicable" mathematics is potentially important. His remarks in this direction are thoughtful and realistic and

work towards demystifying what it is that mathematicians do.

Ellenberg is an engaging and fluid storyteller, and one of the pleasures of the book is the discussion of the mathematical personalities involved in the topics he covers. A particularly attractive aspect of his treatment is an enthusiasm for the compelling stories of mathematical discovery (ranging from satisfying historical melodrama to sheer weirdness) coupled with care to avoid perpetuating a sense that mathematics is the province of "geniuses". In fact, Ellenberg explicitly tries to dispel this notion, and his concrete comments are buttressed by his choice of examples: there is something extremely affirming about the story of the Michigan retirees, Quincy doctors, and MIT undergraduates, all trained to some degree in mathematics but none of them working math-

ematicians, who independently realized how to beat the Massachusetts lottery.

Which brings me to another important running theme of the book: a focus on a fundamentally progressive and egalitarian view of mathematics, a faith that there is a meaningful sense in which both the beauty and the power of mathematics should be and can be accessible to everybody and that this knowledge changes lives for the better. And from a civic standpoint, Ellenberg powerfully makes the case that a key aspect of mathematical literacy for informed citizens is having enough confidence and understanding to be skeptical of conclusions based on mathematical tools employed either incompetently or solely for rhetorical purposes. The only point I might have wished to be emphasized even further is the persistent risk that the effectiveness of mathematics tempts people to use it to obscure inherent uncertainty.

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This is one of the best popular mathematics books I can imagine. There is enough depth for the book to be engaging reading for a working mathematician (especially one without recent formal training in statistics or probability), but of course the real virtue of the book is its value to a nonmathematical audience. It is no exaggeration to say that this book covers as many of the things I would want a numerate nonmathematician to know as a single book of modest length can. There are other topics one could conceivably include, but this is an unimpeachable set, and the style is friendly, warm, and surprisingly funny. Moreover, Ellenberg is aware of the limitations of the medium; he knows full well that no one book is going to change how someone thinks, and thus the text is peppered with the repeated insistence that math can only be learned by practice and that the reader should actually do some math while reading the book.

A possible critique is that the organizational style might seem slightly haphazard: although the chapters link to one another, there isn't a running theme, and earlier topics aren't always reinforced. But I found the structure to be an effective rhetorical choice, especially since many of the topics genuinely do stand on their own. Also, there are a few places (e.g., the discussion of Buffon's needle) where I imagine that a nonmathematician might lose focus (despite the best efforts of the author), but by and large, I suspect Ellenberg's efforts to reach all of his audience will succeed. I think the book will do a service for the mathematical community by explaining what we do and why we do it. And if nothing else, Ellenberg's tone throughout is so reasonable and thoughtful and he seems like such a nice normal guy that he's a good ambassador for the field. It's hard to say that reading this book will necessarily make informed citizens of all of us, but everyone should read it anyway. Even a few more people being just a little bit less wrong would be a good thing. And everyone needs a little more beauty and wonder in their life.

Andrew J. Blumberg in front of his parents' home in Belmont, MA.

Math for the Million\$

In early 2005, MIT math major James M. Harvey was casting about for an independent study project. He started analyzing lotteries, eventually zeroing in on a Massachusetts game called Cash WinFall. After some preliminary research, a conversation with a lottery official confirmed what Harvey suspected: The unusual structure of Cash WinFall guaranteed that, at certain times, a US\$1 bet would on average be worth US\$1.15. Using that fact, Harvey and his fellow MIT alumnus Yuran Lu started buying large numbers of Cash WinFall tickets—and raking in profits. By 2010 they had organized their betting operation into a company, Random Strategies Investments, LLC, named in honor of their MIT dorm Random Hall.

There were other groups of large-scale bettors playing Cash WinFall, including one headed by a medical researcher in Massachusetts and one consisting of a group of retirees in Michigan (where a lottery similar to Cash WinFall had been phased out in 2005). The large-scale betting these groups did was entirely legal and had no adverse impact on the odds of other players winning. The Massachusetts lottery commission knew about the groups' betting and welcomed the increased ticket sales. Nevertheless, a *Boston Globe* investigation in 2011 raised enough questions and controversy to lead to the termination of Cash WinFall.

Cash WinFall differed from most lotteries in that the top prize was capped at US\$2 million. When the jackpot hit \$2 million, the money would be distributed to the lower-tier prizes, in a process called a "roll-down". So for example, matching 4 out of 6 numbers would ordinarily bring a win of US\$150, but during a roll-down, the win became US\$807.52. A 2012 report by Massachusetts Inspector General Gregory W. Sullivan calculated that, during a roll-down, buying US\$400,000 of Cash WinFall tickets had a 50 percent chance of a bringing in US\$425,000 or more.

Jordan Ellenberg's book *How Not to Be Wrong*, and the review here by Andrew Blumberg, give more details about the mathematics of the story (see also "How to Get Rich Playing the Lottery", by *Notices* editorial assistant Katharine Merow, on the website of the Mathematical Association of America, www.maa.org/meetings/calendar-events/how-to-get-rich-playing-the-lottery). Indeed, the math was the most reliable part of the high-scale betting operations. Less reliable were the more mundane aspects: Penciling in betting slips by hand, handling unhappy store owners whose employees were tied up processing thousands of orders, and enduring the vagaries of ticket machines that would jam up in humid weather.

But it was "a lucrative enterprise," Sullivan's report concluded. The report says Harvey's group wagered a total of US\$17–18 million on Cash WinFall and estimates the profits were at least US\$3.5 million before taxes. Mathematics proved that Cash WinFall could be not just a betting game, but a sound investment. Ironically, that's just what led to the game's demise. As Blumberg put it, "Cash WinFall was discontinued when it became clear that it was less of a scam than most lotteries."

—Allyn Jackson