Professor Stewart's *Casebook of Mathematical Mysteries*

Reviewed by Sam Vandervelde

Professor Stewart's Casebook of Mathematical Mysteries

Ian Stewart

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The publication of *Professor Stewart's Cabinet of Mathematical Curiosities* in 2008 established Ian Stewart as a master of an unusual genre of mathematical exposition: one that juxtaposes old puzzles and open problems, that sets whimsical anecdotes and light humor cheek to jowl with forays into recent research. The reader is likely to discover why π is equal to 3 in the Arctic, and also that the shortest opaque fence for a unit circle is believed to equal $\pi + 2$, all in the same sitting. The author stoutly maintains that such a book is properly categorized as a miscellany, and thus is not bound by any particular logical ordering of material, which in turn is not constrained by anything beyond the author's taste. Fortunately, his taste proves to be excellent—not only in the opinion of this reviewer, but also in the eyes of the general public, who bought copies of the original book in droves.

With the wildly successful *Cabinet* seemingly unable to accommodate all the pearls in the good Professor's files, a sequel followed a scant year later. *Professor Stewart's Hoard of Mathematical Treasures* employed the same formula, experimenting with the addition of a pirate theme to enliven some of the posers. With this compilation the

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author was content to rest for a while, but as he writes in the introduction to his latest collection, "They say that three is a good number for a trilogy." And so after a five-year hiatus a third (and final?) volume has arrived, Professor Stewart's *Casebook of Mathematical Mysteries*.

The variety of subject matter, range of sophistication, and abundance of illustrations in this latest installment help to ensure that it will appeal to as wide a readership as ever, from budding middle school math kids to professionals who unflinchingly attend MathFest each summer. (In fact, my fifth grader just stumbled upon the review copy of *Casebook* on my desk and has been deeply absorbed for the past fifteen minutes and counting, pausing only to relay a joke or inform me of the smallest integer that can be written as a sum of two cubes in six different ways.) Granted, Stewart does not hesitate to dress up a number of classic conundrums, such as in *The Scandal of the Stolen Sovereign*, which lays out the familiar puzzle of three diners, a bill that has been reduced from thirty to twenty-five (choose your favorite monetary unit), and a troublesome tip. In other instances favorite chestnuts are presented with no adornment, as is the case for the Jigsaw Paradox in which the same set of pieces are assembled in two configurations to yield regions with purportedly different areas.

However, every reader will find more than enough novel and genuinely delightful mathematical nuggets to make a perusal, if not a careful read, of *Casebook* highly worthwhile. This reviewer was introduced for the first time to the operator theory method of deducing the series for e^x , which involves the observation that $\int e^x = e^x$, hence $(1 - \int)e^x = 0$, leading to $e^x = (1 - \int)^{-1}0$. One now expands $(1 - \int)^{-1}$ as a geometric series, takes successive antiderivatives of 0, sums the results, and the familiar series for e^x appears almost as if

by magic. At the risk of appearing to be poorly read, this reviewer will also confess to being previously ignorant of the fact that there exists a set of three distinct planar regions having precisely the same boundary, not being aware of the latest research explaining why the bubbles in Guinness beer drift downward, and having missed the recent revelation that a single shape (with markings) has been found that tiles a plane but cannot tile it periodically.

There are plenty of tidbits that are accessible to virtually everyone, such as the observation that beginning with the arithmetic identity

$$\begin{aligned} 123789^2 + 561945^2 + 642864^2 \\ = 242868^2 + 761943^2 + 323787^2, \end{aligned}$$

one can successively remove the leftmost digits of all six numbers, yielding a sequence of equalities all of which are valid, concluding with the true statement that $9^2 + 5^2 + 4^2 = 8^2 + 3^2 + 7^2$. In fact, successively removing rightmost digits instead also yields a sequence of equalities, which seems almost too much to ask. However, Stewart does not shy away from more advanced topics, such as an overview of the startling fact that quadratic residues modulo a prime can be used to design efficient sound diffusers for ceilings of concert halls. In another essay he presents the concept of a random harmonic series (in which the sign of each term is chosen randomly) and goes on to survey several quite remarkable characteristics of the resulting probability distribution.

The playful manner in which the Professor presents his mathematical mysteries belies the extreme care with which each is written. From the introduction to the previous volume, we read “*Hoard* is supposed to be fun, not work.” By extension the same may be said of *Casebook*. Indeed, perusing its pages is quite pleasurable, but this is due to the great deal of work expended by the author on our behalf, evident throughout in the clarity of thought, the expansive vocabulary, and the attention to selection and ordering of the various items that Stewart chooses to include. So in the end there is method to the miscellany after all, in the form of optimizing the material to engage and delight as wide an audience as possible. And if the success of *Cabinet* is any indication, Professor Stewart has quite a knack for this sort of optimization.

Approximately one in every four of the essays penned by Ian Stewart in *Casebook* is framed as a memoir of Dr. John Watsup, who embarks upon a series of mathematical adventures while in the company of a certain Hemlock Soames located at 222B Baker Street, across the street from the more well-known pair. (The author assures us at the outset that his concept for this latest compilation of mathematical morsels predates the airing of the BBC series *Sherlock*, which also modernizes this legendary sleuth, albeit in a rather different

manner.) Readers familiar with the works of Sir Arthur Conan Doyle will at once recognize any number of allusions, from Professor Mogiarty to “The Hound of the Basketballs.” The style is also unmistakable; consider this excerpt from “The Affair of the Above-Average Driver,” found on page 169.

The detective sighed. “A common misconception, Watsup.”

“Miscon—what’s wrong with it?”

“Just about everything, Watsup. Suppose one hundred people are assessed on a score ranging from zero to ten. If 99 of them score ten and the other scores zero, what is the average?”

“Uh...990/100...which is 9.9, Soames.”

“And how many are above average?”

“Uh...ninety-nine of them.”

“As I said, a misconception.”

I was not so readily diverted. “But the excess is small, Soames, and the data are not typical.”

“I exaggerated the effect to demonstrate its existence, Watsup. Any data that are skewed—asymmetric—are likely to behave in a similar manner. For example, suppose that most drivers are reasonably competent, a significant minority is appallingly bad, and a very tiny number is excellent. Which drivers are above average in such circumstances?”

“Well...the bad ones bring the average down, and the excellent ones don’t compensate...My word! The competent and excellent drivers are *all* above average!”

This reviewer, who spent countless hours as a child devouring the exploits of Sherlock Holmes, has mixed feelings regarding the use of these beloved characters and distinctive prose to relate matters that at times felt mundane in comparison to the tales on which they are based. However, there is no denying that Stewart is quite adept at emulating Doyle’s style; the reader will be immediately transported to a world in which penetrating deductions based on the barest of clues are related in a casual manner, where our heroes criss-cross London in the late Victorian Era to foil treacherous plots with a mathematical twist, where the game is afoot once again. Although a minority of the pages are devoted to actual storytelling, this style permeates the entire collection, which consequently hangs together in a more or less satisfying way.

Regardless of how the reader feels about this device for bundling math, Stewart brings to the fore an aspect of recounting mathematics that is easily overlooked when assembling problem collections and that is typically expunged from academic journals: Mathematical investigation

unfolds chronologically, with compelling reasons for adopting certain avenues of inquiry, and hence is a narrative. Therefore, it is fitting that mathematics be presented in a narrative format. Of course, the acumen displayed by Detective Soames coupled with Dr. Watsup's admiring enthusiasm serve to draw the reader into the story and therefore into the mathematics as well.

To play devil's advocate for a moment, one could forward the same criticism of Soames and Watsup that has been leveled at the class of textbook word problems reading something like the following. "Charlie is thinking of a right triangle. He tells Deborah that its perimeter is fifty-six and its area is eighty-four. What is the hypotenuse of Charlie's triangle?" The criticism is that packaging mathematics in a social wrapping provides a false motivation for problem-solving. Worse yet, it potentially confuses abstract exercises with real world modeling, a distinct and important field in its own right. Mathematics should be intrinsically compelling; if a topic comes across as uninteresting, the fault lies with its explanation, or in a mismatch on some level between topic and audience. Any number of beloved compilations of mathematical essays rest on the premise that their subject material (and sparkling presentation) is sufficiently arresting to capture and hold the reader's attention—Ross Honsberger springs immediately to mind, for instance.

This review certainly does not intend to settle the debate. But few would deny that mathematics is as much a social as an individual endeavor, or overlook the efficacy of interpersonal communication as a vehicle for motivating and advancing mathematics. And if my fifth grader is any indication, Stewart's approach to incorporating these principles into his latest miscellany is just as powerful as ever.

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