The Logician and the Engineer

Reviewed by Martin Davis

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Paul J. Nahin

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Claude Shannon was my boss when I had a summer job at Bell Labs in Murray Hill, New Jersey, in 1953. Bell Labs in those years had a management that encouraged researchers to follow their own interests. Returning to Bell Labs from a year as a visiting professor at Harvard and MIT, computer scientist Ed Moore wrote, marveling at the pressures to which academic researchers were subject in contrast to his “industrial ivory tower”. The combination of brilliant scientists and this laissez-faire management style certainly paid off, with, for example, seven Nobel Prizes in physics and two Turing Awards in computer science. Among the innovations credited to Bell Labs researchers are the transistor and the laser, the UNIX operating system and the C programming language, and Shannon’s own information theory.

One day during that summer Shannon brought his unicycle to work and rode it down the long corridors as people poured out of the laboratories, shops, and offices. He rode it right into an elevator to repeat the performance on another floor. The same spirit of fun permeated some of his projects. In his office he proudly pointed to a desk calculator he called Throback I, which did arithmetic using Roman numerals. His “mouse” that solved mazes

received national publicity. Another amusing incident that occurred that summer involved the game “matching pennies”. In this game, in each round the two players, A and B, each secretly choose one of the bits 0 or 1. Player A wins if the bits chosen are the same; otherwise B wins. The fun started when one of the engineers, D. W. Hagelbarger, showed us a device he had built to play this game that did a good job of defeating people who didn’t know the algorithm it was implementing. Perhaps a week later, Shannon had his own, much smaller, gadget to play the same game using a different algorithm. One afternoon we all stood around as the two machines were connected to play each other, with the results being tabulated. Shannon’s machine won handily.

The logical operations that underlay Shannon’s gadgets were implemented using electric *relays* (electric switches that are turned on and off by an electric current, producing a magnetic field). AT&T’s telephone network was largely built of these sturdy elements. Shannon’s extremely influential 1938 MIT master’s thesis showed how Boolean algebra could be used to design circuits that carry out any desired logical operations. This thesis changed logical design of electric circuits from an art to routine engineering practice. It is said that Shannon explained this breakthrough as due simply to his happening to know about both Boolean algebra and circuit design. Of course, the crucial step was the realization that the abstract mathematics of logical reasoning and the practical issues of circuit design could be related. Nahin has made the coming together of these distinct strands the focal point of this book, which concerns the lives and work of the two pioneers George Boole and Claude Shannon.

Paul J. Nahin, an electrical engineer who obviously loves mathematics, has written a number of

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books on mathematical subjects that are addressed to a rather general public. He writes in an easy, genial style which, however, makes no effort to avoid technical matters. He certainly ignores the maxim that mathematical formulas scare away potential readers. For example, his charming book *Dr. Euler's Fabulous Formula*,¹ in the course of a long proof that π^2 is irrational, includes the formula

$$R(x) = \frac{1}{n!} \int_0^x (x-s)^n e^s s^n ds.$$

Nahin will begin a book by telling a prospective reader exactly what he expects her/him to know. For example, he writes that a prospective reader of his book on Euler's formula "should have the mathematical background equivalent to what a beginning third year college undergraduate in an engineering or physics program of study will have completed." In the case of the book being reviewed, he asks for much less in the way of mathematical preparation: algebra, including matrix multiplication. But he also expects readers to know Ohm's Law and Kirchhoff's Laws for electric circuits and how to compute the resistance of a pair of resistors in parallel. Nahin completes his prerequisite section by challenging the reader to follow a proof of a result from a paper by Claude Shannon with coauthor Hagelbarger (of the penny-matching machine).

George Boole (1815–1864) was born into the impoverished family of an English cobbler. In addition to showing how certain kinds of logical reasoning can be treated as a branch of algebra,² Boole wrote a paper that may be regarded as founding the theory of invariants, which was so important in nineteenth-century mathematics. He also worked on differential operators and the calculus of finite differences. He was entirely self-educated, later claiming he had chosen mainly mathematics books to study as a money-saving measure because they take longer to finish. The British rigid class system had no real way to deal with a genius like Boole who emerged from the lower middle classes. The problem was solved by exporting Boole to Ireland, where he was appointed to a professorship at a new university that had been founded in Cork. There he prospered, had a

happy marriage, but died of pneumonia when he was only forty-nine years old.

Claude Shannon (1916–2001) was born into a relatively well-to-do Michigan family. His mathematical interests were nurtured by his sister, Catherine, six years his elder, who eventually became a mathematics professor at North Central College in Naperville, Illinois—after obtaining her master's degree in mathematics at the University of Michigan. Although he was to become a highly innovative world-class mathematician, even as a child Claude's passion for making things work led him to do such things as building a radio and a radio-controlled model boat. Shannon's master's thesis in electrical engineering, in which he made the crucial connection between Boolean algebra and switching circuits, was written at MIT under the supervision of Vannevar Bush, who recognized his ability. Shannon had been working with Bush's differential analyzer as his research assistant. Arguably, this analog computer was more powerful than the digital electromechanical large-scale calculators being built at Harvard in the 1930s.

Shannon's 1940 MIT Ph.D. was in mathematics, not engineering, but it was mathematics applied to genetics. After a postdoc year as a National Research Fellow at the Institute for Advanced Study in Princeton, Shannon accepted a position in the mathematics department at Bell Labs, where he was to remain for fifteen very fruitful years. The attack on Pearl Harbor came very soon after he joined the Labs, and he was heavily involved with war work. In 1943 Shannon and Alan Turing, in America for a brief visit, were able to discuss their common interest in cryptography. They also shared an expansive interest in the capabilities of the postwar computers that they expected would be developed. In particular, they discussed the problem of programming such a machine to play chess. Later, Shannon wrote a paper suggesting in outline an algorithm on which such a program might be based.

It was Shannon's attempt to understand the theoretical concepts underlying cryptography that led him to his most remarkable achievement: his development of *information theory* as a full-blown subject. He modeled communication between distant places as involving *messages* sent through a *channel*. A message was conceived of as a sequence of 0s and 1s, and Shannon defined the *information content* or *entropy* of such a message using a formula suggested by that for entropy in statistical thermodynamics. He also defined the *capacity* of a channel in terms of the rate at which data could flow through it. His main theorem showed that, in a suitable sense, data could always be encoded so as to make full use of channel

¹The formula in question is $e^{it} = \cos t + i \sin t$.

²Nahin is not always careful in offhand statements. For example, he says that "Boolean algebra...is also called mathematical logic." I was even more bothered by his statement that Shannon had shown that restricting a Turing machine to two symbols "in no way limits the power of what a Turing machine can do." But this is almost trivial. Shannon did something much more difficult: he showed that limiting a Turing machine to two states is no restriction in what it can do if sufficiently many symbols are available. The casual reader is not likely to be concerned with such matters.

The School of Mathematical Sciences and the Beijing International Center for Mathematical Research at Peking University are actively seeking outstanding candidates for faculty positions in both pure and applied mathematics. Candidates who have demonstrated or shown potential for the highest achievements in all major areas of mathematics are encouraged to apply.

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The discipline of mathematics at Peking University was set up in 1913. Currently, its faculty includes seven members of the Chinese Academy of Sciences as well as many active young mathematicians. The School of Mathematical Sciences has for many years attracted the best undergraduate students in China. The Beijing International Center for Mathematical Research, founded in 2005, has already become a center for attracting top talents as well as international exchanges.

Application materials, including a curriculum vitae, a list of publications, a brief description of current research interest and five main publications, should be addressed to Ms Yang Yang at yangy@math.pku.edu.cn. Junior applicants should arrange three letters of recommendation to be sent directly to Ms Yang. For more information about us, please visit <http://www.math.pku.edu.cn/>, <http://www.bicmr.org/> and <http://hr.pku.edu.cn/rczp/js/>.

capacity. Of particular importance was Shannon's treatment of these matters in the context of a noisy channel. This work was the beginning of an important new field of study with theoretical and practical ramifications involving a number of researchers.

After presenting brief biographies of Boole and Shannon, Nahin introduces Boolean algebra as an algebra of classes and then goes on to show how it can also function as a propositional calculus. A number of logical puzzles are solved using Boolean algebra. Next we are shown how to use Boolean algebra to create switching circuits with electric relays as their basic component—Shannon's key insight. Here Nahin is forced to overstep his announced prerequisite. A pair of switches arranged in a parallel circuit will pass current if either (or both) is in the "on" position, thus implementing logical *or*. But when these switches are relays, turned on by an electric current, it is necessary to incorporate a device that permits current to flow in only one direction in order to avoid a short circuit in which the impulse backs up instead of moving forward. Such a device is called a diode³ and nowadays is implemented in silicon (or similar materials) applying the techniques of solid state physics. Nahin attempts a rather rushed explanation that may not be easy for his readers to follow.

The rest of the book is a potpourri of very brief introductions to various topics: probability theory, error detection and correction, sequential machines capable of being in different "states", Turing machines with Rado's "busy beaver" unsolvable problem, countable and uncountable sets, and, finally, quantum computation.

Paul Nahin has brought together many fascinating things, always presented in a lively manner, usually with care, and with a real effort to provide clear, accurate explanations. This is a book that readers of the *Notices* may well recommend to first-year students or to other young people.

³It is so called because, in the days of vacuum tube electronics, these devices were implemented as vacuum tubes containing just two electrodes.