

ECCENTRICITY OF NETWORKS WITH STRUCTURAL CONSTRAINTS¹

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¹This work was made possible by Partenariat Hubert Curien Proteus [37455VB]; ARRS [BI-FR-PROTEUS/17-18-009, P1-0383], Agence Nationale de la Recherche [anr 10 jcjc 0204 01], the European Commission through the InnoRenew CoE project (Grant Agreement #739574) under the Horizon2020 Widespread-Teaming program and the Republic of Slovenia (Investment funding of the Republic of Slovenia and the European Union of the European regional Development Fund), and the generous support of the Qatar Foundation through Carnegie Mellon University in Qatar's Seed Research program. The statements made herein are solely the responsibility of the authors.

Abstract

The eccentricity of a node v in a network is the maximum distance from v to any other node. In social networks, the reciprocal of eccentricity is used as a measure of the importance of a node within a network. The associated centralization measure then calculates the degree to which a network is dominated by a particular node. In this work, we determine the maximum value of eccentricity centralization as well as the most centralized networks for various classes of networks including the families of bipartite networks (two-mode data) with given partition sizes and tree networks with fixed number of nodes and fixed maximum degree. To this end, we introduce and study a new way of enumerating the nodes of a tree which might be of independent interest.

Keywords: eccentricity, network, bipartite graph, complex network, maximum degree.

2010 Mathematics Subject Classification: Primary: 91D30, Secondary: 05C35, 68R10, 05C05.

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Received 18 April 2017

Revised 18 June 2018

Accepted 23 August 2018