



Catastrophe Progression Method and Its Application to Selection of Construction Scheme

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On the basis of the specific complexity, ambiguity and uncertainty of the selection of construction scheme for multi-attribute decision making system, a new evaluation method based on catastrophe theory is proposed and applied to the decision making for Construction Scheme. Firstly, the multi-layer assessment objects are analyzed with this method and property values of evaluation indexes are quantified. Then, the ultimate catastrophe function is obtained through different catastrophe models. In order to calculate, integrate and obtain the total value of Catastrophe, the decision making can be carried out. The outcome from concrete application of construction scheme optimization indicates that it is reasonable and has avoided the problem of calculating coefficient of weight subjectively with clear thread and simple calculation.

1. Introduction

Selection of Construction Scheme is an important part of optimization in construction, which is restricted by investment, construction period, construction quality, safety, environmental protection, energy policy and other factors. Therefore, in order to achieve better economic efficiency and reduce energy consumption, it is very necessary to compare different schemes in detail. Evaluation index system is large and complex, and the interaction between the evaluation index is common due to the subjectivity of decision-makers and the ambiguity of evaluation index, the interaction affects the decision making results (Zhu and Wu (2015), Li and Zhang (2014)). In recent years, domestic and foreign scholars have carried on a lot of research on the construction scheme optimization methods, and have achieved fruitful results. A lot of decision making methods are proposed and applied, for example multi-criteria decision-making method (Vahid et al (2014), Daniel et al (2014), Ke et al (2012)), simulation and goal programming (Karimi et al (2013), Panagiotis et al (2010), Yang (2014)), TOPSIS method (Rodolfo and Renato (2014), Shu and Liu (2013), Raju and Kumar (2015)), Value Engineering Method (Shu (2014)), Pattern recognition method (Ferreira et al (2014)), Fuzzy matter-element evaluation method (Wang et al (2004)) and Grey method (Zhou and Xiao (2014), Lv and Cui (2002)). It seems that either of these methods is legitimate, but there are also some problems in the mechanism of reflecting the evaluation system (Edmundas et al (2015)). Catastrophe progression evaluation method comes from the catastrophe model, which can be used to solve the multi-attribute decision making problem (Hidekazu et al (2014), Lev and Alexander (2015)). The main characteristics of this method is that it carries out the decomposition of multi-level conflicts on systematic evaluation overall goal at first. Then it brings forward membership function based on catastrophe theory combined with fuzzy mathematics, carrying on integrated quantitative calculations by the normalized formula. At last, it generates a comprehensive parameter and carries out evaluation with the parameter.

2. Catastrophe progression method

Catastrophe theory was put forward by the French mathematician Rene Tom who stemmed from his book of "Structural Stability and Morphogenesis" published in 1972. This book systematically described the catastrophe theory (Antonio (2013)). It studied how nature and human society in a continuous gradient caused Catastrophe or leaps and seeks a unified mathematical model to describe, predict and control the Catastrophe or leap as a new science. Catastrophe theory is a mathematical tool to describe the system transition, giving the parameter region when system is stable and unstable. Accordingly, it proved that when parameters verified, the situation of

system would also change. When the parameters came through some certain specific locations, the state of system will occur Catastrophe.

For a dynamic system, the potential function of the system can be described as:

$$F = F(T, U) \quad (1)$$

F is a collection of system state parameter t_i , $T = \{t_1, t_2 \cdots t_n\}$; U is a collection of system control parameter u_i , $U = \{u_1, u_2 \cdots u_n\}$.

By solving the equations, we can get:

$$\frac{\partial F}{\partial T} = 0, \frac{\partial F}{\partial U} = 0 \quad (2)$$

Get the critical point of system equilibrium state:

$$(T_1, U_1), (T_2, U_2), \cdots (T_n, U_n)$$

Therefore, the center of the catastrophe theory is to study the process characteristics of the system by studying the transformation between the critical points. If the element in the control parameters U is not more than 4, then the function F is at most 7 mutated forms, namely Folding Catastrophe, Peak point Catastrophe, Dovetail Catastrophe, and Butterfly Catastrophe. In the case of Peak point Catastrophe, the catastrophe manifold and bifurcation are shown in Figure 1.

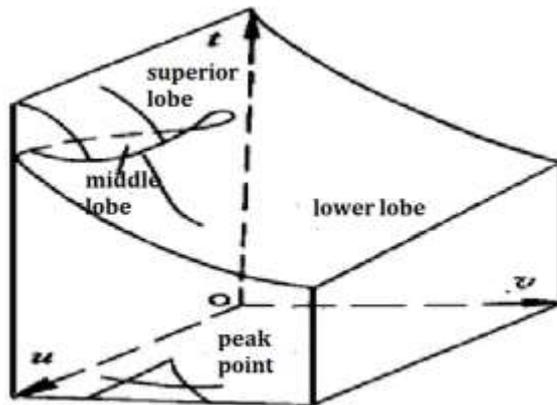


Figure 1: Manifold of Peak point Catastrophe

2.1 Basic model

Table 1: Catastrophe models and normalized equations

order	type	tendency function	dimension of control variable	normalized equation
1	Folding Catastrophe	$V(x) = x^3 + ax$	1	$x_a = \sqrt{a}$
2	Peak point Catastrophe	$V(x) = x^4 + ax^2 + bx$	2	$x_a = \sqrt{a}$, $x_b = \sqrt[3]{b}$
3	Dovetail Catastrophe	$V(x) = \frac{1}{5}x^5 + \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$	3	$x_a = \sqrt{a}$, $x_b = \sqrt[3]{b}$, $x_c = \sqrt[4]{c}$
4	Butterfly Catastrophe	$V(x) = \frac{1}{6}x^6 + \frac{1}{4}ax^4 + \frac{1}{3}bx^3 + \frac{1}{2}cx^2 + dx$	4	$x_a = \sqrt{a}$, $x_b = \sqrt[3]{b}$, $x_c = \sqrt[4]{c}$, $x_d = \sqrt[5]{d}$

The theoretical basis of catastrophe progression method is catastrophe theory, which uses topology theory and singularity theory in dynamic system to build mathematical model. Then the process of qualitative change is

described and forecast when something continuous interruptions happen in natural phenomena and society activity. When state variable is one dimension, there are four Catastrophe models seen from table 1.

There are two kinds of variables in tendency function: (1) state variable x represents behavior state of the system, (2) control variables (a, b, c, d) can be regarded as the factors influencing behavior state. It starts from tendency function, analyses balance curved surface, singular point set and bifurcation set of different Catastrophe models. Then, normalized equation and evaluation standard can be acquired.

Tendency function has two contradictions between state variable and control ones in the system. These variables interact with each other. Hence, every state of system is the unification of state variable and control ones, even of interactions with all control variables.

2.2 Evaluation standard

Facing various practical problems, there are three different kinds of criteria when applying Catastrophe theory to fuzzy composite analysis and evaluation.

(1) Non-complementary criteria: effect of different control variables can't be replaced. That is, they can't complement their shortcomings according to the criteria of min-max.

(2) Complementary criteria: they can make up their deficiency according to mean value.

(3) Complementary over threshold: they can complete after reaching some threshold which risk level can be accepted. Only obeying above criteria can the requirements of bifurcation equation in Catastrophe theory be satisfied.

3. Method of catastrophe progression to optimal scheme

3.1 Construct the system of evaluation indexes

According to the object of optimal Construction Scheme, decompose evaluation objects into some layers from higher to lower. Above indexes are often abstract and not easy to quantify. After decomposition, more concrete indexes can be obtained to quantify readily. When sub-index is calculated, the decomposition will discontinue. Inverted tree hierarchical structure is set up through arraying all indexes. Requirement of Catastrophe evaluation is that more important risk indexes is put frontier and less important ones is behind. In addition, remark that some unimportant indexes should be removed and the layers had better not more than four.

3.2 Normalize decision making matrix

In order to avoid the effect of various dimensions, nondimensionalization of decision making matrix is the first step. Every single index relates to corresponding fuzzy value. Evaluation indexes corresponding to standard scheme belongs to fuzzy membership degree, which is called preferential membership. They are commonly normal. Based on these criteria, such criterion is named preferential membership. In fact, some characters are better when they are larger, while others may be better when smaller. So, different equations are adapted to different membership. However, there are many equations to calculate. To reflect its relativity sufficiently, this paper employs following forms:

The better, the larger

$$\mu_{ij} = X_{ij} / \sum_{j=1}^m X_{ij} \quad (3)$$

The better, the smaller

$$\mu_{ij} = (1 - X_{ij} / \sum_{j=1}^m X_{ij}) / (m - 1) \quad (4)$$

μ_{ij} is preferential membership. m is the number of scheme.

Preferential membership decision making matrix \tilde{R}_{mn} is set up.

$$\tilde{R}_{mn} = \begin{bmatrix} & M_1 & M_2 & \cdots & M_m \\ C_1 & \mu_{11} & \mu_{21} & \cdots & \mu_{m1} \\ C_2 & \mu_{12} & \mu_{22} & \cdots & \mu_{m2} \\ \vdots & \vdots & \vdots & & \vdots \\ C_n & \mu_{1n} & \mu_{2n} & \cdots & \mu_{mn} \end{bmatrix}$$

3.3 Determine the type of catastrophe

When the construction plan is respectively divided into 2, 3 and 4 layers, their corresponding Catastrophe type is peak point, dovetail and butterfly.

3.4 Data integration and evaluation

According to the lowest hierarchy primitive data fuzzy membership of all indexes, decision scheme is selected. Then, use the normalized equation of different Catastrophe systems to composite step by step until the highest layer of construction plan.

4. Case study

The total distance of some channel in the project from south to north is 1840m. The channel is trapezoid section, whose bottom is from 13m to 18m wide and longitudinal shrinking slope is 1/28000. Its first side slope is 1/2~1/3 in the bottom and external slope is 1/1.5~1/1.2. First levee crest is 5.0m wide. Channel slope plate is 10cm thick. Bottom channel is 8cm thick.

4.1 Construct the system of decision making evaluation indexes

On the basis of practical technical level in China construction and Catastrophe evaluation theory, the system of decision making evaluation indexes is constructed. Criteria affecting project construction contain basic demand, technical demand, and social profit and so on. Experts give some score to these quantitative indexes which seen as table 2.

Table 2: Construct the system of decision making evaluation indexes

Object	Criteria	indexes	Attribute of scheme			
			Scheme I	Scheme II	Scheme III	
Optimal scheme of construction	B ₁ basic demand	C ₁ cost/10000yuan	4568	4700	4380	
		C ₂ duration/month	15	17	16	
		C ₃ quality/%	85	82	80	
	B ₂ technical	C ₄ degree of hard/%	80	85	78	
		C ₅ technical extension/%	80	90	75	
		C ₆ management range/%	78	85	70	
		B ₃ social profit	C ₇ social model/%	75	80	72
			C ₈ environmental/%	80	82	78

4.2 Normalize the decision making matrix

According to the equation (3) and (4), normalized decision making matrix is constructed as follows:

$$\tilde{R}_{nm} = \begin{bmatrix} & M_1 & M_2 & M_3 \\ C_1 & 0.331 & 0.348 & 0.321 \\ C_2 & 0.344 & 0.323 & 0.333 \\ C_3 & 0.344 & 0.332 & 0.324 \\ C_4 & 0.335 & 0.325 & 0.340 \\ C_5 & 0.327 & 0.367 & 0.306 \\ C_6 & 0.336 & 0.318 & 0.350 \\ C_7 & 0.330 & 0.352 & 0.317 \\ C_8 & 0.333 & 0.342 & 0.325 \end{bmatrix}$$

4.3 Catastrophe method to integrate and select schemes

For scheme I, integrate the data on the basis of Catastrophe progression evaluation method. Indexes C_1 , C_2 and C_3 form dovetail Catastrophe model based on the complementary criteria,

$$B_1 = (C_1^{1/2} + C_2^{1/3} + C_3^{1/4}) / 3 = (0.331^{1/2} + 0.344^{1/3} + 0.344^{1/4}) = 0.6807 .$$

Indexes C_4 , C_5 , C_6 form dovetail Catastrophe model based on the complementary criteria, Indexes C_7 , C_8 form peak point Catastrophe model based on the complementary criteria,

$$B_3 = (C_7^{1/2} + C_8^{1/3}) / 2 = (0.335^{1/2} + 0.327^{1/3} + 0.333^{1/4}) = 0.6341 .$$

Indexes B_1 , B_2 , B_3 form dovetail Catastrophe model on the basis of min-max criteria,

$$A_1 = \min(B_1^{1/2}, B_2^{1/3}, B_3^{1/4}) = \min(0.6801^{1/2}, 0.6757^{1/3}, 0.6341^{1/4}) = 0.8250 .$$

Based on the similar theory, $A_2 = 0.8235$ for scheme II and $A_3 = 0.8194$ for scheme III.

$A_1 > A_2 > A_3$ so scheme I is the optimal plan.

5. Conclusions

Construct Catastrophe model step by step with the method of Catastrophe progression evaluation. Intergrades all the indexes based on the inner interaction in the normalized equation. Sort indexes in accordance to intrinsic logical relationship to the importance. This can make up the shortcoming of static evaluation method to select plans recently used and reduce subjectivity of assigning weight. In this way, subjective decision making caused by uncertainty can be avoided. Catastrophe progression method is applied to select plans through determine, quantifying and integrating the evaluation model. Evaluation tends to the truth and the result is reasonable. Compared with other methods, Catastrophe evaluation has easy calculation, clear thread and accurate result. Note that there is still something subjectivity in the process of sorting indexes. In addition, normalized equation is integrated. Final composite value is higher and the difference among evaluation values is smaller. Hence, visual effect on evaluation is worse and there is still something to improve in the later research.

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