ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 71, №1, 2018 Механика
УДК 539.3

LOCALIZED BENDING VIBRATIONS OF A RECTANGULAR TWO-LAYER PLATE IN THE PRESENCE OF SLIP BETWEEN LAYERS

Belubekyan M.V., Grishko A.M.

Keywords: two-layered plates, slip, Navier conditions, localized vibrations, free edge.

The problems of vibrations of a plate of two layers are investigated, in the case when the tangential stresses between the layers are zero. Based on the known equations obtained on the basis of Kirchhoff's conjecture, the boundary conditions of the free edge of a rectangular plate are defined. Determined the frequencies of bending vibrations localized in the vicinity of the free edge of the plate.

Բելուբեկյան Մ.Վ., Գրիշկո Ա.Մ.
Երկշերտ ուղղանկյուն սալի կենտրոնացված ծռման տատանումները շերտերի միջև սահքի առկայության դեպքում

Բանալի բառեր. երկշերտ սալ, Նավյեի պայման, կենտրոնացված տատանումներ, ազատ եզր:

Դիտարկվում է երկշերտ սալի տատանումների խնդիրը, երբ շերտերի միջև շոշափող լարումները հավասար են զրոյի: Կիրխգոֆի հիպոթեզի հիման վրա ստացված հայտնի հավասարումներից որոշվում են ուղղանկյուն սալի եզրի եզրային պայմանները: Որոշված են սալի ազատ եզրի շրջակայքում տեղակայված ծռման տատանումների հաճախականությունները:

Белубекян М.В., Гришко А.М.
Локализованные изгибные колебания прямоугольной двухслойной пластины при наличии скольжения между слоями

Ключевые слова: двухслойная пластина, скольжение, условия Навье, локализованные колебания, свободный край.

Исследуются задачи колебаний пластины из двух слоёв в случае, когда между слоями касательные напряжения равны нулю. На основе известных уравнений, полученных на основе гипотезы Кирхгофа, устанавливаются граничные условия свободного края прямоугольной пластины. Определены частоты изгибных колебаний, локализованных в окрестности свободного края пластины.

Introduction. After the fundamental article by Konenkov, about possibility of appearance of localized in the vicinity of the free edge of a plate of bending vibrations, a lot of works on this subject have been published. A review of these works is given in the monograph [2], in a review article [3], in articles [4, 5].

In this paper we discuss the problems of vibrations of two layered plate in case of the tangential stresses between the layers are zero.

1. Statement of the problem. A thin rectangular plate in a rectangular Cartesian coordinate system occupies the region: $0\le x\le a$, $0\le y\le b$, $-h_2\le z\le h_1$. The spatial equations of the theory of elasticity are reduced to the following two-dimensional equations of oscillations of a two-layer plate in the presence of slip between layers based on the assumptions of the Kirchhoff hypothesis in [6, 7]:

$$\Delta u_1+\theta_1\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}\right)-\frac{h_1}{1-\nu_1}\frac{\partial}{\partial x}(\Delta w)=\frac{1}{C_{t1}^2}\left(u_1-\frac{h_1}{2}\frac{\partial w}{\partial x}\right)$$

$$\Delta v_1 + \theta_1 \frac{\partial}{\partial y}\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) - \frac{h_1}{1-\nu_1}\frac{\partial}{\partial y}(\Delta w) = \frac{1}{C_{t1}^2}\left(v_1 - \frac{h_1}{2}\frac{\partial w}{\partial y}\right)$$

$$\Delta u_2 + \theta_2 \frac{\partial}{\partial x}\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right) + \frac{h_2}{1-\nu_2}\frac{\partial}{\partial x}(\Delta w) = \frac{1}{C_{t2}^2}\left(u_2 + \frac{h_2}{2}\frac{\partial w}{\partial x}\right) \tag{1.1}$$

$$\Delta v_2 + \theta_2 \frac{\partial}{\partial y}\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right) + \frac{h_2}{1-\nu_2}\frac{\partial}{\partial y}(\Delta w) = \frac{1}{C_{t2}^2}\left(v_2 + \frac{h_2}{2}\frac{\partial w}{\partial y}\right)$$

$$D\Delta^2 w - \Delta\left[K_1\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) - K_2\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right)\right] + m\frac{\partial^2 w}{\partial t^2} = 0$$

In the derivation of equations (1.1), the plane of contact of the layers of the plate is chosen as the plane z=0. Index (1) refers to a plate with a thickness h_1 ($0<z\le h_1$), index (2) refers to a plate with a thickness h_2 ($-h_2\le z<0$), u_1, v_1 – planar displacements of a plate with an index (1), u_1, v_1 – plate with an index (2), bending functions w(x, y, t) – in accordance with the hypothesis of Kirchhoff. Constants ν_1, ν_2 are Poisson's ratio of materials of the corresponding layers, Δ – two-dimensional Laplace operator. In (1.1) also used the following notations:

$$C_{tk}^2 = \frac{E_k}{2(1+\nu_k)\rho_k},\ \theta_k = \frac{1+\nu_k}{1-\nu_k},\ k = 1, 2$$

$$K_k = \frac{E_k h_k^2}{2(1-\nu_k^2)},\ D = (2/3)*(h_1 K_1 + h_2 K_2),\ m = \rho_1 h_1 + \rho_2 h_2 \tag{1.2}$$

In (1.2) E_k – Young's modules, ρ_k – density of materials of the corresponding layers of the plate.

Some variants of the boundary conditions for the edge of a rectangular plate are also given in [6]. Most clearly these conditions are obtained for the boundary conditions Navier. Suppose that at the edge of a two-layer plate x = const, it is necessary to satisfy the conditions for the normal stress to be zero and the two tangential displacements (with respect to y and z). As a result of the averaging, we obtain:

$$T_1^{(k)} = 0,\ v_k = 0,\ w = 0,\ M_1^{(k)} = 0,\ k = 1, 2 \tag{1.3}$$

In (1.3) $T_1^{(k)}$, $M_1^{(k)}$ – Tensile (compressive) forces and bending moments for the corresponding layers of the plate, which are determined by formulas:

$$T_1^{(k)} = C_k\left[\frac{\partial u_k}{\partial x} + \upsilon_k \frac{\partial v_k}{\partial y} \mp \frac{h_k}{2}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon_k \frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$M_1^{(k)} = K_k\left[\frac{\partial u_k}{\partial x} + \upsilon_k \frac{\partial v_k}{\partial y} \mp \frac{2h_k}{3}\left(\frac{\partial^2 w}{\partial x^2} + \upsilon_k \frac{\partial^2 w}{\partial y^2}\right)\right] \tag{1.4}$$

Where the minus refers to the layer with the index (1), and the plus to the layer with the index (2) and:

$$C_k = \frac{E_k h_k}{1-\nu_k^2} \tag{1.5}$$

Equating (1.4) to zero and using $v_k = 0$, $w = 0$, we obtain:

$$\frac{\partial u_k}{\partial x} \mp \frac{h_k}{2}\frac{\partial^2 w}{\partial x^2} = 0,\ \frac{\partial u_k}{\partial x} \mp \frac{2h_k}{3}\frac{\partial^2 w}{\partial x^2} = 0 \tag{1.6}$$

From system (1.6) follows $\partial u_k/\partial x = 0$, $\partial^2 w/\partial x^2 = 0$ and finally the Navier conditions are reduced to the conditions of hinge plates:

$$\frac{\partial u_k}{\partial x} = 0,\ v_k = 0,\ w = 0,\ \frac{\partial^2 w}{\partial x^2} = 0,\ k = 1, 2 \tag{1.7}$$

Complexities arise, as in the Kirchhoff theory, when boundary conditions are established for the free edge of a plate. The conditions for the stress components of the spatial problem to be zero, after averaging according to Kirchhoff's conjecture, result in the vanishing of the corresponding forces and moments (at x = const)

$$T_1^{(k)} = 0,\ S^{(k)} = 0,\ M_1^{(k)} = 0,\ H^{(k)} = 0,\ N_1^{(k)} = 0\ \ k = 1, 2 \tag{1.8}$$

New efforts ($S^{(k)}$ – shear forces, $N_1^{(k)}$ – transverse shear forces, $H^{(k)}$ – torque) are determined by the formulas:

$$S^{(k)} = \frac{1-\upsilon_k}{2} C_k\left(\frac{\partial u_k}{\partial y} + \frac{\partial v_k}{\partial x} \mp h_k \frac{\partial^2 w}{\partial x \partial y}\right)$$

$$H_1^{(k)} = \frac{1-\upsilon_k}{2} K_k\left(\frac{\partial u_k}{\partial y} + \frac{\partial v_k}{\partial x} \mp \frac{4h_k}{3}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{1.9}$$

$$N_1^{(k)} = \frac{1-\upsilon_k}{2} K_k\left(\Delta u_k + \theta_k \frac{\partial}{\partial x}\left(\frac{\partial u_k}{\partial x} + \frac{\partial v_k}{\partial x}\right)\right) \mp \frac{2h_k}{3} K_k \frac{\partial}{\partial x}\Delta w$$

From the conditions $T_1^{(k)} = 0$, $M_1^{(k)} = 0$, according to (1.4), it follows that

$$\frac{\partial u_k}{\partial x} + \upsilon_k \frac{\partial v_k}{\partial y} = 0,\quad \frac{\partial^2 w}{\partial x^2} + \upsilon_k \frac{\partial^2 w}{\partial y^2} = 0 \tag{1.10}$$

In (1.10), one condition for w is superfluous, it is natural to take the mean instead of the two conditions for w from (1.10):

$$\frac{\partial^2 w}{\partial x^2} + \frac{\upsilon_1 + \upsilon_2}{2}\frac{\partial^2 w}{\partial y^2} = 0 \tag{1.11}$$

What will correspond to the principle of averaging the boundary conditions.

According to Kirchhoff's theory, it is required to combine the conditions of the equality of zero the torque and the transverse shearing force by the general transverse shearing force. In accordance with this, the last two conditions in (1.8) are replaced by the condition:

$$\widetilde{N}_1^{(k)} \equiv N_1^{(k)} + \frac{\partial H^{(k)}}{\partial y} = 0\,,\ k = 1, 2 \tag{1.12}$$

Using condition $S^{(k)} = 0$:

$$\frac{\partial u_k}{\partial x} + \frac{\partial v_k}{\partial y} = \pm h_k \frac{\partial^2 w}{\partial x \partial y} \tag{1.13}$$

The condition that the generalized transverse shearing force is zero ($\widetilde{N}_1^{(k)}$) becomes:

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 w}{\partial x^2} + (2 - \upsilon_k)\frac{\partial^2 w}{\partial y^2}\right] = 0\,,\ k = 1, 2 \tag{1.14}$$

As in the case when the bending moment is zero, an extra condition appears, which, similarly to (1.11), must be replaced by the averaged condition:

$$\frac{\partial}{\partial x}\left[\frac{\partial^2 w}{\partial x^2} + (2 - \frac{\upsilon_1 + \upsilon_2}{2})\frac{\partial^2 w}{\partial y^2}\right] = 0 \tag{1.15}$$

Finally, the conditions for the free edge of a two-layer plate with slip at x = const will be:

$$\frac{\partial u_k}{\partial x} + \upsilon_k \frac{\partial v_k}{\partial y} = 0\,,\ \frac{\partial u_k}{\partial y} + \frac{\partial v_k}{\partial x} = \pm h_k \frac{\partial^2 w}{\partial x \partial y}\ \ k = 1, 2 \tag{1.16}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\upsilon_1 + \upsilon_2}{2}\frac{\partial^2 w}{\partial y^2} = 0\,,\ \frac{\partial}{\partial x}\left[\frac{\partial^2 w}{\partial x^2} + (2 - \frac{\upsilon_1 + \upsilon_2}{2})\frac{\partial^2 w}{\partial y^2}\right] = 0$$

2. General solution of the problem. The system of equations (1.1) can be simplified using the Lame transformation [7]

$$u_k = \frac{\partial \varphi_k}{\partial x} + \upsilon_k \frac{\partial \psi_k}{\partial y}\,,\ v_k = \frac{\partial \varphi_k}{\partial y} - \upsilon_k \frac{\partial \psi_k}{\partial x}\,,\ k = 1, 2 \tag{2.1}$$

Transformations analogous to transformations of the plane problem of the theory of elasticity lead to equations

$$\Delta\psi_k - \frac{1}{C_{lk}^2}\frac{\partial^2 \psi_k}{\partial t^2} = 0\,,\ C_{lk}^2 = \frac{E_k}{(1 - \upsilon_k^2)\rho_k} \tag{2.2}$$

$$\Delta\varphi_k - \frac{1}{C_{lk}^2}\frac{\partial^2 \varphi_k}{\partial t^2} = \pm\frac{h_k}{2}\left(\Delta w - \frac{1}{C_{lk}^2}\frac{\partial^2 w}{\partial t^2}\right) \tag{2.3}$$

$$\Delta^2\left(Dw - K_1\varphi_1 + K_2\varphi_2\right) + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{2.4}$$

Equations (2.2) with respect to planar shear waves in the first and second layers turn out to be autonomous. Equations for longitudinal waves (2.3) and for transverse oscillations turn out to be connected.

We present the formulation of the Konenkov problem [1] for a two-layer plate, in the presence of slip between layers. A semi-infinite plate is considered – strip $0 \le x \le \infty$, $0 \le y \le b$, $-h_2 \le z \le h_1$. The plate consists of two layers $0<z \le h_1$ and $-h_2 \le z<0$, the tangential stresses between them are equal to zero. It is required to find solutions of the system of equations (2.2-2.4) satisfying the Navier boundary conditions (hinge fixing) at edges y = 0; b, boundary conditions of the free edge x = 0 and attenuation conditions:

$$\lim_{x\to\infty} \varphi_k = 0\,,\ \lim_{x\to\infty} \psi_k = 0\,,\ \lim_{x\to\infty} w = 0 \tag{2.5}$$

The boundary conditions of the hinge fixing y = 0; b, similar to conditions (1.7), will have the form:

$$u_k = 0\,,\ \frac{\partial v_k}{\partial y} = 0\,,\ w = 0\,,\ \frac{\partial^2 w}{\partial y^2} = 0 \tag{2.6}$$

The conditions (2.6), after using the transformation (2.1) and some transformations [7], have the following form:

$$\varphi_k = 0\,,\ \frac{\partial \psi_k}{\partial y} = 0\,,\ w = 0\,,\ \frac{\partial^2 w}{\partial y^2} = 0 \qquad y = 0; b \tag{2.7}$$

The new form of writing the boundary conditions for the free edge x = 0 (1.16) will be:

$$\begin{gathered}
\frac{\partial^2 \varphi_k}{\partial x^2} + \upsilon_k \frac{\partial^2 \varphi_k}{\partial y^2} + (1-\upsilon_k)\frac{\partial^2 \psi_k}{\partial x \partial y} = 0 \\
2\frac{\partial^2 \varphi_k}{\partial x \partial y} + \frac{\partial^2 \psi_k}{\partial y^2} - \frac{\partial^2 \psi_k}{\partial x^2} = \pm h_k \frac{\partial^2 w}{\partial x \partial y} \\
\frac{\partial^2 w}{\partial x^2} + \upsilon_c \frac{\partial^2 w}{\partial y^2} = 0\,,\ \text{where}\ \ \upsilon_c = \frac{\upsilon_1 + \upsilon_2}{2} \\
\frac{\partial}{\partial x}\left[\frac{\partial^2 w}{\partial x^2} + (2-\upsilon_c)\frac{\partial^2 w}{\partial y^2}\right] = 0
\end{gathered} \tag{2.8}$$

Thus, it is required to find a solution of the system of equations (2.2) - (2.4) satisfying the boundary conditions (2.2), (2.8) and the damping condition (2.5). The existence of such a solution means the existence of oscillations localized in a neighborhood of the free edge.

The solutions of the system of equations (2.2), (2.4) satisfying the hinge-binding conditions (2.7) can be represented in the form:

$$\begin{gathered}
\psi_k = e^{i\omega t}\sum_{n=0}^{\infty} \psi_{kn}(x)\cos\lambda_n y\,,\quad \lambda_n = \frac{\pi n}{b} \\
\varphi_k = e^{i\omega t}\sum_{n=1}^{\infty} \varphi_{kn}(x)\sin\lambda_n y \\
w = e^{i\omega t}\sum_{n=1}^{\infty} w_n(x)\sin\lambda_n y
\end{gathered} \tag{2.9}$$

The substitution of (2.9) into (2.2-2.4) leads to the following system of ordinary differential equations:

$$\psi''_{kn} - \lambda_n^2(1-\xi_k^2)\psi_{kn} = 0$$

$$\varphi''_{kn} - \lambda_n^2(1-\theta_k\xi_k^2)\varphi_{kn} = \pm\frac{h_k}{2}\left[w''_n - \lambda_n^2\left(1-\theta_k\xi_k^2\right)w_n\right] \tag{2.10}$$

$$D\left(w_n^{IV} 2\lambda_n^2 w''_n + \lambda_n^4 w_n\right) - K_1\left(\varphi_{1n}^{IV} - 2\lambda_n^2\varphi''_{1n} + \lambda_n^4\varphi_{1n}\right) +$$

$$+K_2\left(\varphi_{2n}^{IV} - 2\lambda_n^2\varphi''_{2n} + \lambda_n\varphi_{2n}\right) - m\omega^2 w_n = 0$$

where $\xi_k^2 = \dfrac{\omega^2}{\lambda_n^2 C_{tk}^2}$, $\theta_k = \dfrac{C_{tk}^2}{C_{lk}^2}$ (2.11)

The solution of system (2.10) is represented in the form:

$$\psi_{kn} = F_k e^{-\lambda_n px},\ \varphi_{kn} = B_k e^{-\lambda_n px},\ w_n = Ae^{-\lambda_n px} \tag{2.12}$$

so that the positive roots of the characteristic equation with respect to p satisfy the damping conditions (2.5).

Substituting (2.12) into system (2.10) leads to algebraic equations with respect to arbitrary constants F_k, B_k, A:

$$\left(p^2 - 1 + \xi_k^2\right)F_k = 0$$

$$\left(p^2 - 1 + \theta_k\xi_k^2\right)\left(B_k \mp \frac{h_k}{2}A\right) = 0 \tag{2.13}$$

$$\left(p^2 - 1\right)^2\left(DA - K_1B_1 + K_2B_2\right) - \frac{m\omega^2}{\lambda_n^4}A = 0$$

3. The system (2.13) admits various variants of obtaining the characteristic equation of the problem. Let's consider the case:

$$p^2 \neq 1 - \xi_k^2,\ p^2 \neq 1 - \theta_k\xi_k^2 \tag{3.1}$$

In this case we obtain from (2.13)

$$F_k = 0,\ B_k = \pm\frac{h_k}{2}A \tag{3.2}$$

Taking into account the expression for B_k from (3.2) in the third equation of the system (2.13) we obtain the characteristic equation

$$\left(p^2 - 1\right)^2 - \eta^2 = 0, \tag{3.3}$$

where

$$\eta^2 = \frac{m\omega^2}{\lambda_n^4 D_1},\ D_1 = \frac{1}{6}\left(h_1K_1 + h_2K_2\right) \tag{3.4}$$

Equations (3.3) coincide with the characteristic equation of the Konenkov problem. The influence of the two-layered plate is included in the dimensionless parameter η^2, characterizing the phase velocity roots of equation (3.3) satisfying the damping requirement will be

$$p_1 = \sqrt{1+\eta}\,,\; p_2 = \sqrt{1-\eta} \tag{3.5}$$

In this case, additional conditions are necessary:

$$0 < \eta < 1 \tag{3.6}$$

The final solution for the deflection function will be:

$$w = e^{i\omega t}\sum_{n=1}^{\infty}\left(A_{1n}e^{-\sqrt{1+\eta}\lambda_n x} + A_{2n}e^{-\sqrt{1-\eta}\lambda_n x}\right)\sin\lambda_n y \tag{3.7}$$

The substitution of (3.7) into the boundary conditions of the free edge (2.8) leads to a system of homogeneous algebraic equations with respect to A_{1n}, A_{2n}. The equation that determines the dimensionless frequency of localized bending vibrations is obtained from the condition that the determinant of this system is equal to zero [9].

$$p_1^2 p_2^2 + 2\left(1-\upsilon_c\right)p_1 p_2 - \upsilon_c^2 = 0 \tag{3.8}$$

The difference from the Konenkov equation is that instead of the Poisson ratio, the mean value of the Poisson coefficients of the two layers is here. This circumstance may be important for anisotropic materials, or if there are materials with a negative Poisson's ratio.

Another difference between the problem considered here – the excitation of localized bending vibrations leads to the appearance of localized longitudinal oscillations:

$$\varphi_{kn} = \mp\frac{h_k}{2}\left(A_{1n}e^{-\sqrt{1+\eta}\lambda_n x} + A_{2n}e^{-\sqrt{1-\eta}\lambda_n x}\right) \tag{3.9}$$

Conclusion. The problems of localized bending vibrations of a rectangular two-layered plate are considered in case of slip between layers. Under the conditions taken into account in this paper, the Navier condition coincides with the hinging. The boundary conditions of the free edge of a rectangular plate are obtained. A comparison of this problem with the Konenkov problem is given. The difference from the Konenkov equation is that instead of the Poisson ratio, the mean value of the Poisson coefficients of the two layers is in the equation that determines the dimensionless frequency of localized bending vibrations. This circumstance may be important for anisotropic materials, or if there are materials with a negative Poisson's ratio. Another difference between the problem considered here –localized bending vibrations leads to the appearance of localized longitudinal oscillations.

ЛИТЕРАТУРА

1. Коненков Ю.К. Об изгибной волне рэлеевского типа. // Акуст. журнал.1960. Т.6. №1. С.124-126. Konenkov Yu.K. A Rayleigh-Type Flexural Wave // Soviet Physics. Acoustics. 1960. Vol.6, iss. 1. P. 122–123.
2. Вильде М.В., Каплунов Ю.Д., Коссович Л.Ю. Краевые и интерфейсные резонансные явления в упругих телах. // М.: Физматлит, 2010. 280с. Wilde M. V., Kaplunov J.D., Kossovich L.Yu. Edge and interfacial resonance phenomena in elastic bodies. // Moscow: Fizmatlit. 2010. 280 p.
3. Laurie J.B., Kaplunov J.D. Edge waves and resonance on elastic structures. a overview. // Mathematics and Mechanics of solids. 2012. V.17. №1. P.4-16.
4. Belubekyan M., Ghazaryan K., Marzoca P. Localized Flexural Vibrations of Membrane Cylindrical Shell. // The Journal of the Acoustical Society of America. 2017. 141(3). P. 1947-1952.

5. Belubekyan M. V. On the condition of Planar Localized Vibration Appearance in the Vicinity of the free Edge of Thin Rectangular Plate. // Proceedings of the Yerevan State University. (Physical and Mathematical Sciences) 2017. 51(1). P. 42-45.
6. Белубекян М.В. Изгиб и колебания двухслойной пластинки при условиях свободного скольжения между слоями. // Изв. НАН Армении. Механика. 2013. Т.66. №2. С.14-24. Belubekyan M.V. Bending and oscillations of a two-layer plate under conditions of free sliding between layers. // NAS of Armenia. Mechanics. 2013. 66 (2), p.14-24.
7. Гришко А.М. Изгиб двухслойной пластинки при условии свободного скольжения между ортотропными слоями. // Механика – 2016: Труды международной школы-конференции молодых учёных, 3-7 октября 2016, Цахкадзор, Армения, Ереван: Ереванский государственный университет архитектуры и строительства. 2016. 198с. Grishko A. M. The bending of a two-layer plate under the condition of free sliding between orthotropic layers. // Mechanics 2016: Proceedings of International School-Conference of Young Scientists. 3-7 October, 2016, Tsakhkadzor, Armenia, p.198.
8. Новацкий В. Теория упругости. // Перев. с польск. Б.Е. Победри. – М.: Мир, 1975. 256с. OCR-слой. Novacky W. Theory of elasticity. // Trans. from polish. Pobedri B.E. – M.: Mir, 1975. 256 p.
9. Амбарцумян С.А., Белубекян М.В. К вопросу об изгибных волнах, локализованных вдоль кромки пластинки// Прикладная механика НАН Украины. 1994. Т.30. №2. С.61-68. Ambartsumian S.A., Belubekyan M. V. To the problem of bending waves localized along the edge of a plate // Applied mechanics, NAS Ukraine, 1994, 30 (2), p.61-68.

<u>**Сведения об авторах:**</u>

Белубекян Мелс Вагаршакович – кандидат физ.-мат. наук, профессор, главный научный сотрудник Института механики НАН Армении, Ереван, Армения.
Тел: (+374 10) 521503, (+374 10) 580096. **E-mail:** mbelubekyan@yahoo.com

Гришко Анна Микаеловна – аспирантка, каф. механики, ЕГУ, фак-т матем. и механики, Ереван, Армения. **Тел:** (+374 98) 152820,
E-mail: annochka1986@gmail.com

Поступила в редакцию 05.09.2017