Jestr

Journal of Engineering Science and Technology Review 9 (4) (2016) 77 - 81

Research Article

JOURNAL OF
Engineering Science and Technology Review

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Design of Fractional-Order Filters Using Current Feedback Operational Amplifiers

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Received 30 June 2015; Accepted 15 January 2016

Abstract

The design of fractional-order filters using Current Feedback Operational Amplifiers as active elements, is presented in this paper. The first step of the procedure is the approximation of the fractional-order transfer function by an appropriate integer-order function and the derivation of the corresponding Functional Block Diagram. As a second step, the implementation of the derived Functional Block Diagram is performed through the utilization of integration and summation blocks constructed from Current Feedback Operational Amplifiers. As a design example, a 1+*a* order (0<*a*<1) lowpass filter has been realized and its behaviour has been evaluated through experimental results using the commercially available AD844 discrete IC component as Current Feedback Operational Amplifier.

Keywords: Analog filters, Fractional-order filters, Current Feedback Operational Amplifiers.

1. Introduction

The growing research interest for employing the concept of fractional calculus in electronic engineering is mainly originated from the interdisciplinary nature of this research area. For example, the modeling of viscoelasticity as well as of biological cells and tissues has been performed through the utilization of the fractional-order calculus. Biological signals such as electrocardiograms (ECG) and electroencephalographs (EEG) have spectra that do not increase or decrease by multiples of ±6 dB/octave but by multiples of $\pm 6\cdot a$ dB/octave ($0<a<1$). In addition, the capability for precisely controlling the attenuation gradient in fractional-order filters in comparison with the corresponding integer-order filters is an attractive feature.

Fractional Order Elements (FOEs) are the main building blocks for performing signal processing according to the fractional calculus. Unfortunately, these elements are not commercially available and, thus, FOEs are approximated by appropriately configured RC networks. Following this approach a number of voltage-mode filters where Operational Amplifiers (Op-Amps), second-generation Current Conveyors (CCIIs), and Current Feedback Operational Amplifiers (CFOAs) are employed as active elements have been proposed in the literature. A drawback of these topologies is the employment of floating resistors and capacitors [1-12].

Another solution for realizing fractional-order filters is the approximation of their transfer function by a suitable integer-order filter function [13-17]. In [13] the realization of fractional order filters has been performed by employing a cascade connection of first and second-order filters sections which were available through a Field Programmable Analog Array (FPAA). As a result, the resulted filter topologies will suffer from increased sensitivity with respect to the effect of component variations. An alternative approach has been introduced in [14], where the fractional-order filters have been implemented through a cascade of first and second-order filter sections realized via parallel RC network and Single Amplifier Biquad (SAB).

A systematic design procedure for designing voltage-mode fractional-order filters is presented in this paper. The utilized active cell is the Current Feedback Operational Amplifier (CFOA), due to its versatility and design flexibility. As a result, the most attractive feature of the resulted filter topologies is the requirement for only grounded resistors and capacitors. The paper is organized as follows: the design procedure is presented in a systematic way in Section 2, while the derived fractional-order filter topology using CFOAs as active elements is given in Section 3. The behaviour of the proposed filter is evaluated, through experimental results, using the commercially available AD844 discrete IC component in Section 4.

2. Design procedure for fractional-order filters

2.1. Lowpass fractional filters with order 1+*a* (0<*a* <1)

According to the analysis provided in [13-14], the direct realization of a fractional filter of order $n+a$ is stable only in the case that $n+a<2$. Therefore, only $1+a$ order fractional filters realizations offer stability. The transfer function of a fractional lowpass filter is given by (1) as:

$$H_{1+a}^{LP}(s)=\frac{K_1}{s^{1+a}+K_2} \tag{1}$$

The low-frequency gain is equal to K_1/K_2, and the half-power frequency is given by (2) as

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ISSN: 1791-2377

$$\omega_{hp} = \left[K_2\left(\sqrt{1+\cos^2\frac{(1+a)\pi}{2}} - \cos\frac{(1+a)\pi}{2}\right)\right]^{\frac{1}{1+a}} \quad (2)$$

The obtained frequency responses suffer from the presence of an undesired peaking equal to $K_1/\{K_2\cdot[(\sin(1+a)\pi/2]\}$at frequency $\omega_p=\{-K_2\cdot[\cos(1+a)\pi/2]\}^{-(1+a)}$. In order to overcome this problem, the modified transfer function given by (3) will be employed, which intends to approximate the all-pole Butterworth response by introducing an extra term equal to K_3s^a in the denominator of the transfer function in (1)

$$H_{1+a}^{LP}(s) = \frac{K_1}{s^{1+a} + K_3 s^a + K_2} \quad (3)$$

The factors K_i in (3) are given by the expressions in (4), which are derived through an appropriate algorithm for minimizing the errors in the frequency response [13]

$$K_1 = 1 \quad (4a)$$

$$K_2 = 0.2937a + 0.71216 \quad (4b)$$

$$K_3 = 1.068a^2 + 0.161a + 0.3324 \quad (4c)$$

An efficient approximation of the term s^a, with regards to circuit complexity, is the second-order expression given by (5), which is derived according to the Continued Fraction Expansion (CFE) formula [8], [13]

$$s^a \cong \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \quad (5)$$

The expressions for the terms a_0, a_1, a_2 are given by (6)

$$\begin{cases} a_0 = a^2 + 3a + 2 \\ a_1 = 8 - 2a^2 \\ a_2 = a^2 - 3a + 2 \end{cases} \quad (6)$$

Substituting (6) into (3) the transfer function of the lowpass filter with Butterworth characteristics becomes as in (7)

$$H_{1+a}^{LP}(s) \cong \frac{K_1}{a_0}\frac{a_2 s^2 + a_1 s + a_0}{s^3 + b_2 s^2 + b_1 s + b_0} \quad (7),$$

where

$$b_0 = \frac{a_0 K_2 + a_2 K_3}{a_0} \quad (8a)$$

$$b_1 = \frac{a_1(K_2 + K_3) + a_2}{a_0} \quad (8b)$$

$$b_2 = \frac{a_1 + a_0 K_3 + a_2 K_2}{a_0} \quad (8c),$$

and the values of a_o, a_1, a_2 are determined by (6).

A way for realizing the integer-order transfer function in (7), which approximates a fractional-order filter of order 1+a, is using the Functional Block Diagram (FBD) of the Follow the Leader Feedback (FLF) topology depicted in Fig. 1.The transfer function is given by (9) as

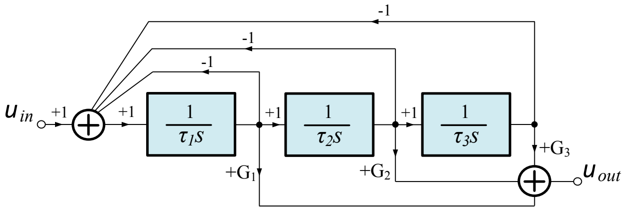

Fig. 1. FBD for realizing a fractional filter of order 1+a

$$H(s) = \frac{\frac{G_1}{\tau_1}s^2 + \frac{G_2}{\tau_1\tau_2}s + \frac{G_3}{\tau_1\tau_2\tau_3}}{s^3 + \frac{1}{\tau_1}s^2 + \frac{1}{\tau_1\tau_2}s + \frac{1}{\tau_1\tau_2\tau_3}} \quad (9)$$

Comparing the coefficients of the corresponding terms in (7) and (9) it is readily obtained that

$$\tau_1 = \frac{1}{b_2} \quad (10a)$$

$$\tau_2 = \frac{b_2}{b_1} \quad (10b)$$

$$\tau_3 = \frac{b_1}{b_0} \quad (10c)$$

and

$$G_1 = \frac{K_1 a_2}{a_0 b_2} \quad (11a)$$

$$G_2 = \frac{K_1 a_1}{a_0 b_1} \quad (11b)$$

$$G_3 = \frac{K_1}{b_0} \quad (11c)$$

The expressions given by (10) and (11) will be used, at circuit level realization, for the calculation of the values of the elements of the filter.

In order to facilitate the reader, the design procedure for a lowpass/highpass filter of the order 1+a will be summarized in an algorithmic way. Thus, the steps that should be followed are:

- *Step#1*: Determination of the value of a.
- *Step#2*: Calculation of the values of factors K_i (i=2, 3), according to (4).
- *Step#3*: Calculation of the values of coefficients a_i (i=0, 1, 2), according to (6).
- *Step#4*: Calculation of the values of coefficients b_i (i=0, 1, 2), according to (8).
- *Step#5*: Calculation of the values of time-constants τ_i (i=1, 2, 3), according to (10).
- *Step#6*: Denormalization of the values calculated in step#5 at the desired frequency.
- *Step#7*: Calculation of the values of scale factors G_i (i=1, 2, 3) according to (11).

2.2. Lowpass fractional filters of order *n*+*a* (0<*a* <1)

The realization of a fractional lowpass filter of the order n+a with Butterworth characteristics is performed using the polynomial ratio given by (12)

$$H_{n+a}^{LP}(s) = \frac{H_{1+a}^{LP}(s)}{B_{n-1}(s)} \quad (12)$$

where $H_{1+a}^{LP}(s)$ is the transfer function given by (7) and $B_{n-1}(s)$ is the corresponding Butterworth polynomial of order n-1 [13].

Using the expression in (7), then from (12) it is obtained the general form given by (13)

$$H_{n+a}^{LP}(s) \cong \frac{K_1}{a_o} \cdot \frac{a_2 s^2 + a_1 s + a_0}{s^{n+2} + c_{n+1} s^{n+1} + \cdots + c_1 s + c_0} \tag{13}$$

where the coefficients c_k (k=0,1,..,n+1) are defined by the values of b_i (i=0,1,2) and the coefficients of the polynomial $B_{n-1}(s)$.

A general FBD for the implementation of (13) is demonstrated in Fig. 2.

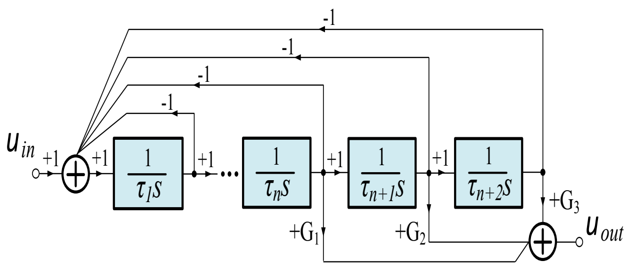

Fig. 2. FBD for realizing a fractional filter of order n+a

The realized transfer function is

$$H_{n+a}^{LP}(s) = \frac{\frac{G_1}{\tau_1\tau_2...\tau_n} s^2 + \frac{G_2}{\tau_1\tau_2...\tau_{n+1}} s + \frac{G_3}{\tau_1\tau_2...\tau_{n+2}}}{s^{n+2} + \frac{1}{\tau_1} s^{n+1} + \frac{1}{\tau_1\tau_2} s^n + \cdots + \frac{1}{\tau_1\tau_2...\tau_{n+2}}} \tag{14}$$

Comparing the coefficients of (13) and (14) it is derived that, under the assumption that c_{n+2}=1, the time-constants are calculated by the formula in (15) as

$$\tau_j = \frac{c_{n+3-j}}{c_{n+2-j}} \quad j = 1,2,...,n+2 \tag{15}$$

while the corresponding gain factors are given by the expressions in (16)

$$G_1 = \frac{K_1 a_2}{a_0 c_2} \tag{16a}$$

$$G_2 = \frac{K_1 a_1}{a_0 c_1} \tag{16b}$$

$$G_3 = \frac{K_1}{c_0} \tag{16c}$$

In order to formulate the procedure for designing a high-order fractional lowpass filter, the required steps are the following:

- *Step#1*: Determination of the value of a.
- *Step#2*: Calculation of the values of factors K_i (i=2, 3), according to (4).
- *Step#3*: Calculation of the values of coefficients a_i (i=0, 1, 2), according to (6).
- *Step#4*: Calculation of the values of coefficients b_i (i=0, 1, 2), according to (8).
- *Step#5*: Derivation of the transfer function according to (12).
- *Step#6*: Calculation of the coefficients c_k (k=0,1,..,n+1).
- *Step#7*: Calculation of the values of time-constants τ_i (i=1, 2, 3), according to (15).
- *Step#8*: Denormalization of the values calculated in step#7 at the desired frequency.
- *Step#9*: Calculation of the values of scale factors G_i (i=1, 2, 3), according to (16).

It should be mentioned at this point that an alternative way for designing a n+a order filter could be the cascade connection of a 1+a order filter with a n-1 order filter. This achieved facilitation of the design procedure is achieved at the expense of the sensitivity of the whole filter.

3. Fractional-order filters using CFOAs

The realization of the FBD in Fig. 1, using CFOAs as active elements, is depicted in Fig. 3.

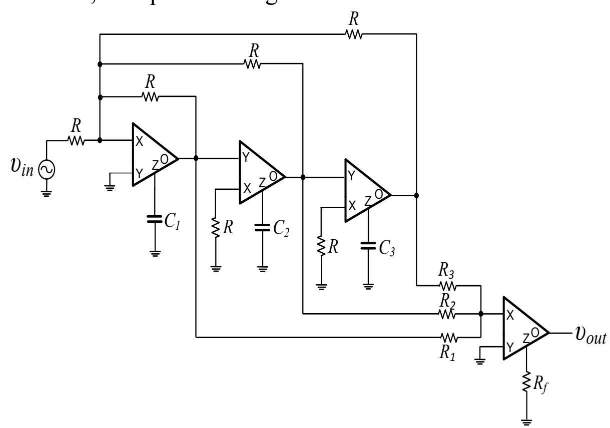

Fig. 3. Fractional filter of order 1+a using CFOAs as active elements

In a similar way, the FBD in Fig. 2 could be realized using CFOAs. In both realizations the time-constants as well as the gain factors are given by the formulas: τ_i=RC_i and G_i=R_f/R_i (i=1,2,3), respectively. Therefore, using the aforementioned formulas and the expressions in (10)-(11) and (15)-(16) a fractional filter of arbitrary order could be designed.

Inspecting the topology in Fig.3 it is readily obtained that only grounded capacitors and resistors are required. This is very attractive feature from the integration point of view, because the effect of parasitics in high frequency applications is minimized. In addition, resistors can be emulated by appropriately configured active elements offering electronic tuning capability of the cutoff frequency of the filter as well as of the order of the filter. In this way, the filter topology could be adopted to the now days trend for designing analog filter with electronically programmable characteristics and without using conventional passive resistors. Another important feature of the filter in Fig.3 is originated from the fact that output is derived at the terminal O of the corresponding CFOA. Taking into account that this is the output of the internal buffer of the CFOA it is concluded that the structure in Fig.3 is suitable for direct cascade connection; in other words, the employment of additional buffers is avoided leading to reduction of the active component count in comparison with the corresponding structures where CCIIs are employed.

4. Experimental results

The behavior of the filter in Fig.3 is evaluated through experimental results, using the commercially available AD844 discrete IC components as CFOAs, biased at ±10V. Using the design step presented in Section 2, and considering that R=10KΩ, the calculated values of the passive components for realizing fractional lowpass filters with Butterworth characteristics and cutoff frequency 10kHz are summarized in Table 1.

The experimental setup is demonstrated in Fig. 4. It should be mentioned at this point that DIP switches have been utilized in order to achieve programmability of the order of the filter

Table 1. Element values of the filter in Fig.3.

Element	Order 1.1	Order 1.5	Order 1.8
R (kΩ)	10	10	10
R_1 (kΩ)	58.8	142.8	529.8
R_2 (kΩ)	13.1	16.3	21.3
R_3 (kΩ)	10	10	10
R_f (kΩ)	10	10	10
C_1 (nF)	0.37	0.56	0.63
C_2 (nF)	1.53	1.38	1.41
C_3 (nF)	7.22	5.24	4.5

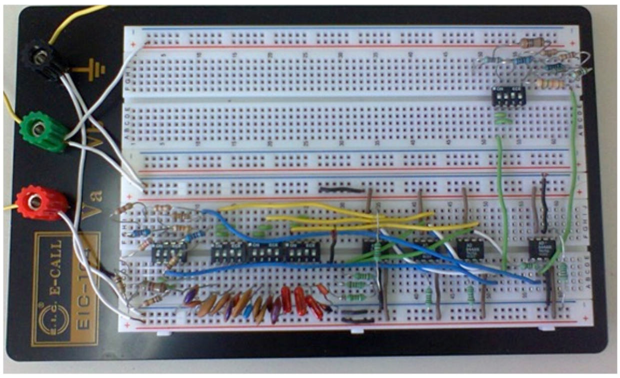

Fig. 4. Experimental setup for evaluating the filter in Fig.3

The measured frequency response of a 1.5 order filter is demonstrated in Fig.5, where the cutoff frequency was 9.3kHz while the slope of the stopband attenuation was equal to -9.5dB/oct. Taking into account that the corresponding theoretically predicted values were 10kHz and -9dB/oct, respectively, the correct operation of the filter in Fig.3 is verified. The observed deviations are mainly caused by the effect of parasitics of AD844 as well as by the tolerances of the used passive resistors and capacitors. These can be easily compensated through appropriate trimming.

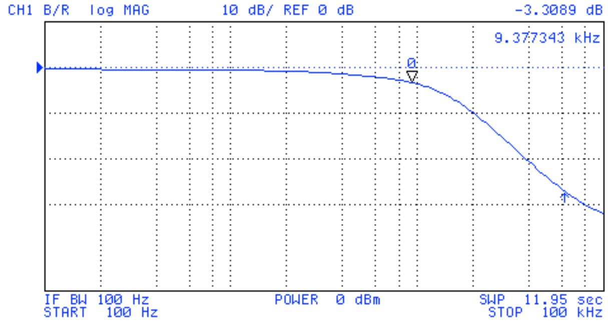

Fig. 5. Frequency response of the filter in Fig.3 in the case of 1.5 order.

5. Conclusion

Fractional-order filters could be designed in a systematic way following the procedure presented in this paper. The employment of CFOAs as active elements offers the benefit of realizations with only grounded passive elements and capability for cascade connection without the requirement of extra buffering stages. The provided experimental results, where AD844 discrete IC components have been used as CFOAs, confirm the correct operation of the proposed fractional-order filter structure in terms of cutoff frequency as well as of the attenuation gradient. Thus, the proposed realization could be considered as an attractive candidate for realizing fractional-order filters using discrete IC components.

This paper was presented at Pan-Hellenic Conference on Electronics and Telecommunications - PACET, that took place May 8-9 2015, at Ioannina Greece.

Acknowledgment

This work was supported by Grant E.029 from the Research Committee of the University of Patras (Programme K. Karatheodori).

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