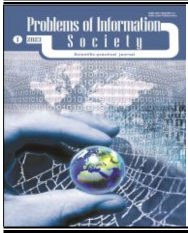

www.jpis.az

14 (1)
2023

Volatile time series forecasting on the example of the dynamics of the Dow Jones index

Ramin R. Rzayev[1], Parvin E. Alizada[2], Tahir Z. Mehdiyev[1]

[1,3]Ministry of Science and Education Republic of Azerbaijan Institute of Control Systems, B. Vahabzadeh str., 9, Baku, Azerbaijan

[2]Baku State University, Z. Khalilov str., 23, Baku, Azerbaijan

[1]raminrza@yahoo.com, [2]palizade@inbox.ru, [3]tahir.mehdiyev@gmail.com

[1]*orcid.org/0000-0001-7658-2850*, [2]*orcid.org/0000-0002-4699-7868*, [3]*orcid.org/0000-0002-9350-0578*

ARTICLE INFO

http://doi.org/10.25045/jpis.v14.i1.02

Article history:
Received 01 September 2022
Received in revised form
04 November 2022
Accepted 10 January 2023

Keywords:
Dow Jones industrial average
Fuzzy time series
Fuzzy set
Fuzzy inference
Neural network
Aproximation

ABSTRACT

The paper discusses a new predictive model of a fuzzy volatile time series, in the framework of which a new approach to the data fuzzification is proposed as the results of observations based on "Soft Measurements". As an example, the index of the Dow Jones Industrial Average is chosen, the readings of which are established by usual arithmetic averaging of contextual indicators. This allows to consider the daily readings of the Dow Jones index as weakly structured, and to interpret the dynamics of its change as a fuzzy time series. The data fuzzification is realized by applying the fuzzy inference system that provides the values of the membership functions of the appropriate fuzzy sets on the universe covering the set of Dow Jones index for the period from June 15, 2018 to October 10, 2019. The proposed predictive model is based on the identified internal relationships, designed as 1st order fuzzy relations between evaluation criteria (or fuzzy sets) that describe weakly structured Dow Jones indexes. At the end of the study, the proposed model is evaluated for adequacy using the statistical criteria MAPE, MPE and MSE.

1. Introduction

Over the past three decades, numerous publications have been devoted to the study of fuzzy time series, among which are the works by Q. Song and B. Chissom [1, 2], N. Kumar et al. [3], S.M. Chen [4, 5], S.M. Chen et al. [6-10], C. Cheng et al. [11], J. Poulsen [12], K. Huarng [13], Y. Huang et al. [14], V. Uslu et al. [15], etc. The approaches existing in these works differ in the rules of data fuzzification and defuzzification of fuzzy forecasts as outputs of the corresponding fuzzy models. In order to address the shortcoming of traditional forecasting methods, Song and Chissom [1, 2] first defined the Fuzzy Time Series (FTS) forecasting method for predicting university acceptance rates in 1993. The original FTS structure is very complex and requires a lot of computing resources. It has been replaced by a more efficient Chen model [4, 5], which is generally accepted by researchers and is a common form of FTS. Chen [4] proposed a simple calculation method to get higher prediction accuracy in the FTS modeling. Until now, this model is used as the basis for FTS modeling.

The reliability of the final fuzzy predicts depends on how these rules adequately describe the weakly structured data by appropriate fuzzy sets and, accordingly, interpret fuzzy predicts in a traditional numerical manner. In [16], on the example of the Dow Jones Industrial Average (DJIA) we propose methods for data fuzzification using a mechanism of fuzzy inference. However, the tasks of forming qualitative criteria for evaluating the indicators of the DJIA and the fuzzy model of the corresponding time series remain unresolved. Therefore, on the basis of the same example, this paper proposes a new method for the formation of a sufficient set of evaluation criteria and building an appropriate predictive model on this basis.

2. Problem Definition

The object of the study is the DJIA time series, covering the set of index indicators for the period from June 15, 2018 to October 10, 2019 (Fig. 1) [17]. The DJIA index is established by daily trading on the US stock exchange using the arithmetic averaging of its constituent indicators. Therefore, each value of the DJIA index x_t at time t can be considered as a weakly structured historical data, which can be interpreted by an appropriate fuzzy set A_k ($k = 1 \div m$) characterized by following tuple:

$$\{x(t) / \mu_{A_k}(x_t)\},\ \mu_{A_k}(x_t) \to [0,1].$$

It is necessary to develop a data fuzzification method for the DJIA index, which would allow to adequately restore the time series in terms of fuzzy sets and, thereby, increase the reliability of its forecasting by appropriate fuzzy model. The developed predictive model must be checked for adequacy by means of applying the statistical evaluation criteria, such as Mean Squared Error (MSE), the Mean Absolute Percentage Error (MAPE) and the Mean Percentage Error (MPE).

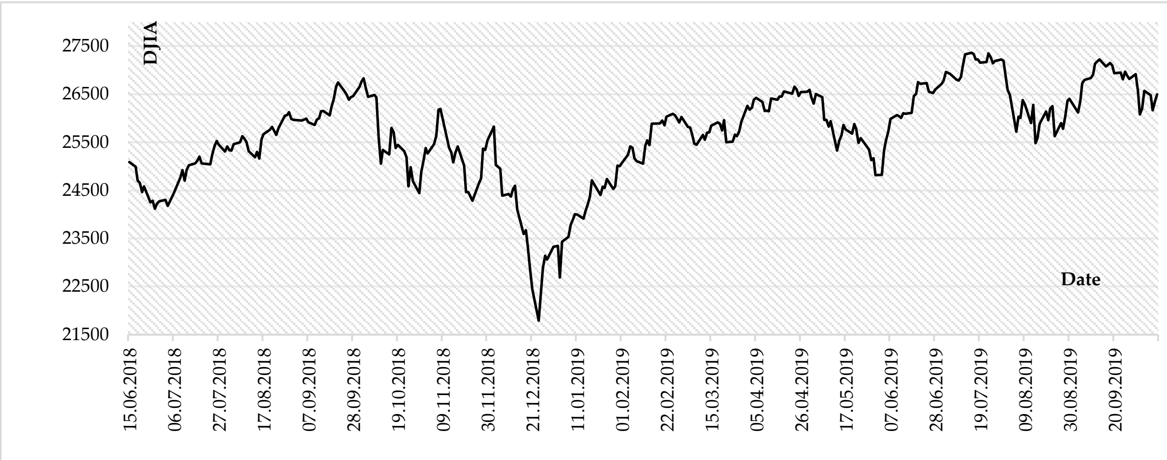

Fig. 1. DJIA index Time Series

3. DJIA Index Fuzzification

To fuzzify DJIA indexes the segment D=[D_{min}-D_1, D_{max}+D_2] is chosen as the universe, where D_{min} and D_{max} are the minimum and maximum values of the DJIA, respectively; D_1>0 and D_2>0 are selected based on the division of the segment D into equal intervals u_j according to the number of qualitative evaluation criteria. Suppose that such criteria are 8 terms of the linguistic variable "DJIA index value", which are represented as the following appropriate fuzzy sets on the universe $U = \{u_1, u_2, \ldots, u_8\}$:

- TOO LOW: $C_1 = \mu_{C_1}(u_1)/u_1 + \ldots + \mu_{C_1}(u_8)/u_8$;
- VERY LOW: $C_2 = \mu_{C_2}(u_1)/u_1 + \ldots + \mu_{C_2}(u_8)/u_8$;
- MORE THAN LOW: $C_3 = \mu_{C_3}(u_1)/u_1 + \ldots + \mu_{C_3}(u_8)/u_8$;
- LOW: $C_4 = \mu_{C_4}(u_1)/u_1 + \ldots + \mu_{C_4}(u_8)/u_8$;
- HIGH: $C_5 = \mu_{C_5}(u_1)/u_1 + \ldots + \mu_{C_5}(u_8)/u_8$;
- MORE THAN HIGH: $C_6 = \mu_{C_6}(u_1)/u_1 + \ldots + \mu_{C_6}(u_8)/u_8$;
- VERY HIGH: $C_7 = \mu_{C_7}(u_1)/u_1 + \ldots + \mu_{C_7}(u_8)/u_8$;
- TOO HIGH: $C_8 = \mu_{C_8}(u_1)/u_1 + \ldots + \mu_{C_8}(u_8)/u_8$,

where $\mu_{C_j}(u_i) \in [0,1]$ $(j,i = 1,2,\ldots,8)$ are the value of the membership function representing the belonging of the interval u_i to the fuzzy set C_j. In other words, if the index DJIA belongs to the interval u_i, then it is described by the appropriate qualitative criterion (or corresponding fuzzy set) C_j.

In the considered time series, which includes 333 indicators of DJIA index, D_{min}=21792.2 and D_{max}=27359.2. Choosing D_1 = 21.2 and D_2 = 11.8, a coverage in the form of the segment U = [21771, 27371] is obtained. According to the chosen number of quality evaluation criteria C_i, this segment is divided into eight equal intervals of 700 units long: u_1=[21771, 22471], u_2=[22471, 23171], u_3=[23171, 23871], u_4=[23871, 24571], u_5=[24571, 25271], u_6=[25271, 25971], u_7=[25971, 26671], u_8=[26671, 27371]. In this case, historical data is interpreted as fuzzy set C_j, taking into account that the interval of its localization u_j (j=1÷8) belongs to C_j. To do this, we choose the following consistent and rather trivial statements as a basis:

e_1: "If the DJIA index is located closer to the middle of the segment u_1, then its value is too low";

e_2: "If the DJIA index is located closer to the middle of the segment u_2, then it is very low";

e_3: "If the DJIA index is located closer to the middle of the segment u_3, then it is more than low";

e_4: "If the DJIA index is located closer to the middle of the segment u_4, then its value is low";

e_5: "If the DJIA index is located closer to the middle of the segment u_5, then its value is high";

e_6: "If the DJIA index is located closer to the middle of the segment u_6, then its value is more than high";

e_7: "If the DJIA index is located closer to the middle of the segment u_7, then it is very high";

e_8: "If the DJIA index is located closer to the middle of the segment u_8, then it is too high".

The analysis of these information fragments makes it possible to identify one input characteristic in the form of a linguistic variable (LV) x="*Data localization*", the values of which are the terms: "CLOSER TO THE MIDDLE OF THE SEGMENT u_i" (i = 1÷8), and one output LV y="*DJIA value*" with terms: TOO LOW, VERY LOW, MORE THAN LOW, LOW, HIGH, MORE THAN HIGH, VERY HIGH, TOO HIGH.

The verbal assessment of the localization of the $x(t)$ on the basis of belonging to one or another segment u_j (j=1÷8) is represented as a fuzzy subset of the universe, consisting of all DJIA index: $U=\{x(t)\}_{t=1}^{333}$. As a membership function, the following function of the Gaussian type is chosen:

$$\mu(x)=\exp[-(x_t-u_{j0})^2/\sigma^2], \qquad (1)$$

where x_t=$x(t)$ is the DJIA index obtained as a result of the completion of trading on the stock exchange for the t-th day; u_{j0} is the middle of the interval u_j (j=1÷8); σ^2 is a variation, which is chosen as the same for all cases, as 2500000 (Fig. 2).

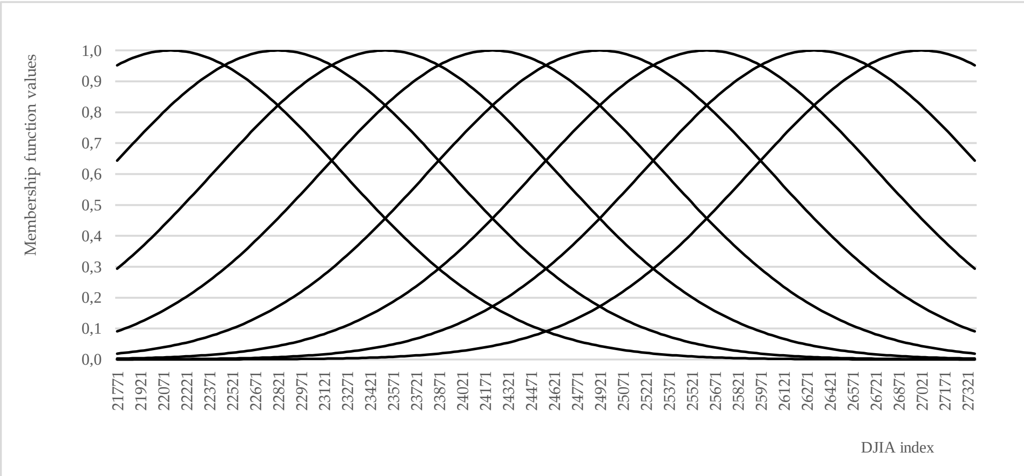

Fig. 2. Gaussian Membership Functions Reflecting the Degree of DJIA Localization

Noting that the midpoints of the segments u_j are the corresponding numbers: u_{10}=22121, u_{20}=22821, u_{30}=23521, u_{40}=24221, u_{50}=24921, u_{60}=25621, u_{70}=26321, u_{80}=27021, according to (1) feature of each $x(t)$ localization are interpreted by following fuzzy sets [18]:

- "CLOSE TO u_{10}": X_1 = 0.9522/x_1 + 0.9586/x_2 + 0.9646/x_3 +…+ 0.0008/x_{332} + 0.0005/x_{333};
- "CLOSE TO u_{20}": X_2 = 0.6434/x_1 + 0.6569/x_2 + 0.6703/x_3 + … +0.0069/x_{332} + 0.0045/x_{333};
- "CLOSE TO u_{30}": X_3 = 0.2938/x_1 + 0.3042/x_2 + 0.3147/x_3 + … + 0.0411/x_{332} + 0.0290/x_{333};
- "CLOSE TO u_{40}": X_4 = 0.0906/x_1 + 0.0952/x_2 + 0.0999/x_3 + … + 0.1643/x_{332} + 0.1260/x_{333};
- "CLOSE TO u_{50}": X_5 = 0.0189/x_1 + 0.0201/x_2 + 0.0214/x_3 + … + 0.4439/x_{332} + 0.3704/x_{333};
- "CLOSE TO u_{60}": X_6 = 0.0027/x_1 + 0.0029/x_2+ 0.0031/x_3 + … + 0.8104/x_{332} + 0.7358/x_{333};
- "CLOSE TO u_{70}": X_7 = 0.0003/x_1 + 0.0003/x_2 + 0.0003/x_3 + …+0.9998/x_{332} + 0.9877/x_{333};
- "CLOSE TO u_{80}": X_8 = 0.000016/x_1 + 0.000018/x_2 + 0.000020/x_3 + … + 0.8334/x_{332} + 0.8959/x_{333}.

The terms of the output LV y can be described by appropriate fuzzy sets on the universe I = {0, 0.1, 0.2, …; 1}. So, according to [18] $\forall i\in I$ we have followings:

- TL=TOO LOW, $\mu_{TL}(i)=\begin{cases}0, & i=1,\\ 1, & i<1;\end{cases}$
- VL=VERY LOW: $\mu_{VL}(i)=(1-i)^2$;
- ML=MORE THAN LOW: $\mu_{ML}(i)=\sqrt{1-i}$;
- L=LOW: $\mu_L(i)=1-i$;
- H=HIGH: $\mu_H(i)=i$;
- MH=MORE THAN HIGH: $\mu_{MH}(i)=\sqrt{i}$;
- VH=VERY HIGH: $\mu_{VH}(i)=i^2$;
- TH=TOO HIGH, $\mu_{TH}(i)=\begin{cases}1, & i=1,\\ 0, & i<1.\end{cases}$

Taking into account the introduced formalisms, the above verbal model e_1 ÷ e_8 can be realized by the fuzzy inference system, which in symbolic form is as follows:

e_1: $(x = X_1) \Rightarrow (y = TL)$; e_2: $(x = X_2) \Rightarrow (y = VL)$;
e_3: $(x = X_3) \Rightarrow (y = ML)$; e_4: $(x = X_4,) \Rightarrow (y = L)$;
e_5: $(x = X_5) \Rightarrow (y = H)$; e_6: $(x = X_6) \Rightarrow (y = MH)$;
e_7: $(x = X_7) \Rightarrow (y = VH$»; e_8: $(x = X_8) \Rightarrow (y = TH)$.

After transforming these rules using, for example, Lukasiewicz's fuzzy implication:

$$\mu_W(u,i) = \min\{1, 1 - \mu_X(u) + \mu_Y(i)\},$$

for each pair $(u, i) \in X \times Y$ on $X \times Y$ fuzzy relations are formed [8]. In particular, for the 1st rule, the fuzzy relation is represented as the following matrix

$$R_1 = \left[\begin{array}{c|ccccccccccc} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0.0294 & 1.0000 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 & 0.9706 \\ 0.0374 & 1.0000 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 & 0.9626 \\ 0.0699 & 1.0000 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 & 0.9301 \\ M & M & M & M & M & M & M & M & M & M & M & M \\ 0.0008 & 1.0000 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 & 0.9992 \\ 0.0005 & 1.0000 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 & 0.9995 \end{array}\right]$$

As a result of the intersection of all fuzzy relations, a general functional solution $R = R_1 \cap R_2 \cap \ldots \cap R_8$ is obtained in the form of the following matrix

$$R = \left[\begin{array}{c|ccccccccccc} & 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\ \hline x_1 = 25090.5 & 0.0114 & 0.1114 & 0.2114 & 0.3114 & 0.4114 & 0.5114 & 0.6114 & 0.5610 & 0.4610 & 0.3610 & 0.2610 \\ x_2 = 24987.5 & 0.0018 & 0.1018 & 0.2018 & 0.3018 & 0.4018 & 0.5018 & 0.6018 & 0.5094 & 0.4094 & 0.3094 & 0.2094 \\ x_3 = 24700.2 & 0.0193 & 0.1193 & 0.2193 & 0.3193 & 0.4193 & 0.5193 & 0.4878 & 0.3878 & 0.2878 & 0.1878 & 0.0878 \\ M & M & M & M & M & M & M & M & M & M & M & M \\ x_{332} = 26346.0 & 0.0002 & 0.0102 & 0.0402 & 0.0902 & 0.1602 & 0.1666 & 0.1666 & 0.1666 & 0.1666 & 0.1666 & 0.8357 \\ x_{333} = 26496.7 & 0.0123 & 0.0223 & 0.0523 & 0.1023 & 0.1041 & 0.1041 & 0.1041 & 0.1041 & 0.1041 & 0.1041 & 0.8740 \end{array}\right]$$

which reflects the cause-effect relations between the localization features of historical data x_t and the estimates of the DJIA index. In other words, the fuzzy set A_t reflects the t-th DJIA index on the universe I, the corresponding values of the membership function of which are located on the t-th row of the matrix R. In particular, the analogue of the historical data x_1=25090.5 is the fuzzy set (1st row of matrix R):

$$A_1 = \frac{0.0114}{0} + \frac{0.1114}{0.1} + \frac{0.2114}{0.2} + \frac{0.3114}{0.3} + \frac{0.4114}{0.4} + \frac{0.5114}{0.5} + \frac{0.6114}{0.6} + \frac{0.5610}{0.7} + \frac{0.4610}{0.8} + \frac{0.3610}{0.9} + \frac{0.2610}{1}$$

Thus, all DJIA index are represented in the form of corresponding fuzzy sets as presented in Table 1.

Table 1. Detailed Fuzzy Analogue of the DJIA Time Series

Date	DJIA	FS	Values of the corresponding membership function											PE FS
			0,0000	0,1000	0,2000	0,3000	0,4000	0,5000	0,6000	0,7000	0,8000	0,9000	1,0000	
06/15/2018	25090,5	A_1	0,0114	0,1114	0,2114	0,3114	0,4114	0,5114	0,6114	0,5610	0,4610	0,3610	0,2610	0,6062
06/18/2018	24987,5	A_2	0,0018	0,1018	0,2018	0,3018	0,4018	0,5018	0,6018	0,5094	0,4094	0,3094	0,2094	0,5939
06/19/2018	24700,2	A_3	0,0193	0,1193	0,2193	0,3193	0,4193	0,5193	0,4878	0,3878	0,2878	0,1878	0,0878	0,5330
……														
12/21/2018	22445,4	A_{132}	0,9138	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0226
12/24/2018	21792,2	A_{133}	0,9801	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0216
……														
10/09/2019	26346,0	A_{332}	0,0002	0,0102	0,0402	0,0902	0,1602	0,1666	0,1666	0,1666	0,1666	0,1666	0,8357	0,9321
10/10/2019	26496,7	A_{333}	0,0123	0,0223	0,0523	0,1023	0,1041	0,1041	0,1041	0,1041	0,1041	0,1041	0,8740	0,9534

The last column of Table 1 presents point estimates of fuzzy sets (PE FS) or, which is the same, defuzzified values of the corresponding FS A_t (t=1÷333), which conditionally restore the configuration of the DJIA time series on the scale of the segment [0, 1] (Fig. 3).

PE FS are established according to the following reasoning. For a fuzzy subset of the universal discrete set, i.e. in our case, for $A \subset I$ the α-level sets are defined as follows: $A_\alpha = \{i \mid \mu_A(i) \geq \alpha, i \in I\}$ ($\alpha \in [0, 1]$). Further, for each set A_α, the corresponding cordinal number $M(A_\alpha)$ is calculated according to the formula:

$$M(A_\alpha) = \sum_{k=1}^{n} u_k / n, \; u_k \in A_\alpha. \tag{2}$$

As a result, PE of the fuzzy set A is established by the formula:

$$F(A) = (1/\alpha_{\max}) \int_0^{\alpha_{\max}} M(A_\alpha) d\alpha \tag{3}$$

where $\alpha_{\max}$ is the maximal value on the fuzzy set A.

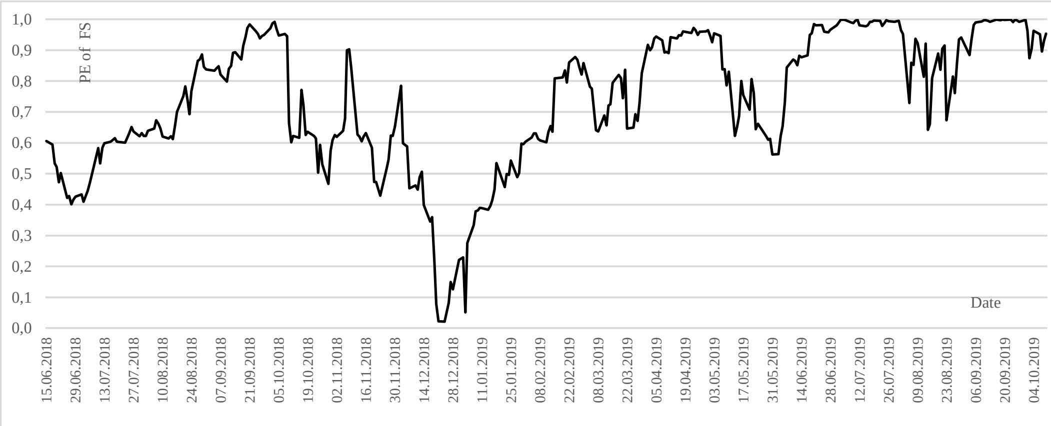

Fig. 3. DJIA Time Series in the Notation of PE

So, according to (2) for the fuzzy set A_1 as analogue of the DJIA index x_1, we have followings:

- $0< \alpha <0.0114$, $\Delta\alpha = 0.0114$, $A_{1\alpha} = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1\}$, $M(A_{1\alpha}) = 0.50$;
- $0.0114< \alpha <0.1114$, $\Delta\alpha = 0.1$, $A_{1\alpha} = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1\}$, $M(A_{1\alpha}) = 0.55$;
- $0.1114< \alpha <0.2114$, $\Delta\alpha = 0.1$, $A_{1\alpha} = \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1\}$, $M(A_{1\alpha}) = 0.60$;
- $0.2114< \alpha <0.2610$, $\Delta\alpha = 0.0495$, $A_{1\alpha} = \{0.3, 0.4, 0.5, 0.6, 0.7, 0.9, 1\}$, $M(A_{1\alpha}) = 0.65$;
- $0.2610< \alpha <0.3114$, $\Delta\alpha = 0.0505$, $A_{1\alpha} = \{0.3, 0.4, 0.5, 0.6, 0.7, 0.9\}$, $M(A_{1\alpha})=0.60$;
- $0.3114< \alpha <0.3610$, $\Delta\alpha = 0.0495$, $A_{1\alpha} = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$, $M(A_{1\alpha})=0.65$;
- $0.3610< \alpha <0.4114$, $\Delta\alpha = 0.0505$, $A_{1\alpha} = \{0.4, 0.5, 0.6, 0.7, 0.8\}$, $M(A_{1\alpha}) = 0.60$;
- $0.4114< \alpha <0.4610$, $\Delta\alpha = 0.0495$, $A_\alpha = \{0.5, 0.6, 0.7, 0.8\}$, $M(A_{1\alpha}) = 0.65$;
- $0.4610< \alpha <0.5114$, $\Delta\alpha = 0.0505$, $A_{1\alpha} = \{0.5, 0.6, 0.7\}$, $M(A_{1\alpha}) = 0.60$;
- $0.5114< \alpha <0.5610$, $\Delta\alpha = 0.0495$, $A_{1\alpha} = \{0.6, 0.7\}$, $M(A_{1\alpha}) = 0.65$;
- $0.5610< \alpha <0.6114$, $\Delta\alpha = 0.0505$, $A_{1\alpha} = \{0.6\}$, $M(A_{1\alpha})= 0.60$.

Then, according to (3), the PE of the fuzzy set A_1 is calculated as follows:

$$F(A_1) = (1/0.6114)\int_0^{0.6114} M(A_\alpha)d\alpha = [0.0114\cdot 0.5 + 0.1\cdot 0.55 + 0.1\cdot 0.60 + 0.0495\cdot 0.65 + 0.0505\cdot 0.60 + 0.0495\cdot 0.65 + 0.0505\cdot 0.60 + 0.0495\cdot 0.65 + 0.0505\cdot 0.60 + 0.0495\cdot 0.65 + +0.0505\cdot 0.60]/0.6114 = 0.6062.$$

The fuzzy sets presented in Table 1 form an overly redundant set of qualitative evaluation criteria, which does not allow the use of well-known models for fuzzy time series forecasting. Therefore, it is necessary to establish the appropriate number of fuzzy sets. To do this, we use the step-by-step procedure proposed in [19].

Step 1. Sort the DJIA index x_t (t=1÷333) into an ascending sequence $\{x_{p(i)}\}$, where p is a permutation that sorts the DJIA values in ascending order: $x_{p(i)} \le x_{p(i+1)}$.

Step 2. Calculate the average value over the totality of all pairwise distances $d_i = |x_{p(i)}-x_{p(i+1)}|$ between any two consecutive values $x_{p(t)}$ and $x_{p(t+1)}$ according to the formula:

$$AD = \frac{1}{n-1}\sum_{i=1}^{n-1}|x_{p(i)} - x_{p(i+1)}| \qquad (4)$$

and standard deviation according to the formula

$$\sigma_{AD} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1}(d_i - AD)^2} \qquad (5)$$

Step 3. Eliminate the anomalies, that is, outliers that need to be removed. The values of pairwise distances that do not satisfy the condition are subject to removed:

$$AD - \sigma_{AD} \le d_i \le AD + \sigma_{AD}. \qquad (6)$$

Step 4. Recalculate AD on the set of pairwise distances remaining after their resorting and, assuming $D_1=D_{\min}-AD$, $D_2=D_{\max}+AD$, calculate the appropriate m number of context fuzzy sets according to the formula

$$m = (D_2 - D_1 - AD) / (2\cdot AD). \qquad (7)$$

Thus, guided by formulas (4) and (5), for the DJIA time series (n=333), we have: AD=16.7681 and σ_{AD}=43.8484. Removing d_i that do not satisfy the condition

$$-27.0803 \approx 16.7681 - 43.8484 \le d_i \le 16.7681 + 43.8484 \approx 60.6165,$$

the final value of the average value for the totality of the remaining pairwise distances d_i is obtained as AD=9.9740. Then, choosing the universe as segment $D=[D_{min}-AD,\ D_{max}+AD]=[D_1,\ D_2]$ ([10]), where D_1=21792.2-9.9740=21782.2260, D_2=27359.2+ +9.9740=27369.1740, according to the formula (7), the appropriate number of FS, reflecting the qualitative criteria for evaluating the DJIA index, are established as:

$$m = (27369.1740\text{-}21782.2260\text{-}9.9740) / (2\cdot 9.9740) = 279.5765.$$

Further, assuming F_1 as point estimate of FS A_1 and F_{333} as point estimate of FS A_{333}, the segment [F1, F333] is divided into 280 equal segments a_k (k=1÷280) with length (F_{333}-F_1)/280. Then the FSs A_t (t=1÷333) (Table 1) can be distributed among the corresponding groups by the rule: "If PE of FS A_t from the interval a_k (k=1÷280), then A_t is included in the k-th group". As a result of this distribution, 144 groups are obtained, within which qualitative evaluation criteria are formed and summarized in Table 2. The remaining 136 groups are empty, i.e., do not include PE of any fuzzy sets.

Table 2. Qualitative Criteria for Evaluating the DJIA Index

Criteria	Values of the membership function of the fuzzy set on the universe I										
	0,0000	0,1000	0,2000	0,3000	0,4000	0,5000	0,6000	0,7000	0,8000	0,9000	1,0000
C_1	0,9138	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412
C_2	0,8644	0,1200	0,1200	0,1200	0,1200	0,1200	0,1200	0,0972	0,0472	0,0172	0,0072
C_3	0,8173	0,1960	0,1960	0,1960	0,1960	0,1960	0,1606	0,0906	0,0406	0,0106	0,0006
C_4	0,8115	0,2051	0,2051	0,2051	0,2051	0,2051	0,1613	0,0913	0,0413	0,0113	0,0013
C_5	0,7489	0,2985	0,2985	0,2985	0,2985	0,2730	0,1830	0,1130	0,0630	0,0330	0,0230
C_6	0,7193	0,3392	0,3392	0,3392	0,3392	0,2896	0,1996	0,1296	0,0796	0,0496	0,0396
C_7	0,6396	0,4393	0,4393	0,4393	0,4393	0,3461	0,2561	0,1861	0,1361	0,1061	0,0155
C_8	0,6379	0,4414	0,4414	0,4414	0,4414	0,3475	0,2575	0,1875	0,1375	0,1075	0,0149
C_9	0,6292	0,4514	0,4514	0,4514	0,4514	0,3545	0,2645	0,1945	0,1445	0,1145	0,0121
C_{10}	0,5875	0,4978	0,4978	0,4978	0,4978	0,3892	0,2992	0,2292	0,1792	0,1492	0,0031
……											
C_{135}	0,0291	0,0391	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,8946
C_{136}	0,0369	0,0469	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,9014
C_{137}	0,0440	0,0516	0,0516	0,0516	0,0516	0,0516	0,0516	0,0516	0,0516	0,0516	0,9067
C_{138}	0,0422	0,0422	0,0422	0,0422	0,0422	0,0422	0,0422	0,0422	0,0422	0,0422	0,9800
C_{139}	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,9173
C_{140}	0,0283	0,0283	0,0283	0,0283	0,0283	0,0283	0,0283	0,0283	0,0283	0,0283	0,9215
C_{141}	0,0117	0,0217	0,0217	0,0217	0,0217	0,0217	0,0217	0,0217	0,0217	0,0217	0,8730
C_{142}	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,9297
C_{143}	0,0091	0,0091	0,0091	0,0091	0,0091	0,0091	0,0091	0,0091	0,0091	0,0091	0,9351
C_{144}	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,9423

If the group includes only one fuzzy set, then this set forms the evaluation criterion. In cases where the group includes two or more fuzzy sets, the evaluation criterion is formed in the form of their intersection using the "min" operation on the corresponding values of the membership functions. For example, the 1st group includes fuzzy sets A_{132} and A_{133}, whose PE are located in the interval [0.0216, 0.0251] (Table 1). Information about these sets on the universe I, including the values of their membership functions and PE, calculated by formulas (2) – (3), as well as relative to the fuzzy set $C_1=A_{132}\cap A_{133}$, as the 1st criterion for evaluating the DJIA index, are summarized in the Table 3.

As another example, a group of fuzzy sets is chosen (Table 4), PE of which are located in the last 144-th interval [0.9960, 0.9995]. Here, the qualitative criterion for evaluating the DJIA index is formed as following intersection:

$C_{144} = A_{264}\cap A_{265}\cap A_{269}\cap A_{275}\cap A_{279}\cap A_{311}\cap A_{312}\cap A_{315}\cap \cap A_{316}\cap A_{317}\cap A_{318}\cap A_{319}\cap A_{320}\cap A_{322}\cap A_{323}\cap A_{325}$.

Table 3. Formation of the C_1 Criterion for Evaluating the DJIA Index

FS	Values of membership functions of fuzzy sets from the 1st group											PE FS
	0,0000	0,1000	0,2000	0,3000	0,4000	0,5000	0,6000	0,7000	0,8000	0,9000	1,0000	
A_{132}	0,9138	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0226
A_{133}	0,9801	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0423	0,0216
C_1	0,9138	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0226

Table 4. Formation of the C_{144} Criterion for Evaluating the DJIA Index

FS	Values of membership functions of fuzzy sets from the 144-th group											PE FS
	0,0000	0,1000	0,2000	0,3000	0,4000	0,5000	0,6000	0,7000	0,8000	0,9000	1,0000	
A_{264}	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,9509	0,9994
A_{265}	0,0039	0,0039	0,0039	0,0039	0,0039	0,0039	0,0039	0,0039	0,0039	0,0039	0,9460	0,9979
A_{269}	0,0018	0,0018	0,0018	0,0018	0,0018	0,0018	0,0018	0,0018	0,0018	0,0018	0,9627	0,9991
A_{275}	0,0071	0,0071	0,0071	0,0071	0,0071	0,0071	0,0071	0,0071	0,0071	0,0071	0,9680	0,9963
A_{279}	0,0057	0,0057	0,0057	0,0057	0,0057	0,0057	0,0057	0,0057	0,0057	0,0057	0,9670	0,9970
A_{311}	0,0050	0,0050	0,0050	0,0050	0,0050	0,0050	0,0050	0,0050	0,0050	0,0050	0,9445	0,9974
A_{312}	0,0054	0,0054	0,0054	0,0054	0,0054	0,0054	0,0054	0,0054	0,0054	0,0054	0,9667	0,9972
A_{315}	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,0012	0,9617	0,9994
A_{316}	0,0032	0,0032	0,0032	0,0032	0,0032	0,0032	0,0032	0,0032	0,0032	0,0032	0,9646	0,9983
A_{317}	0,0063	0,0063	0,0063	0,0063	0,0063	0,0063	0,0063	0,0063	0,0063	0,0063	0,9674	0,9967
A_{318}	0,0022	0,0022	0,0022	0,0022	0,0022	0,0022	0,0022	0,0022	0,0022	0,0022	0,9632	0,9989
A_{319}	0,0029	0,0029	0,0029	0,0029	0,0029	0,0029	0,0029	0,0029	0,0029	0,0029	0,9475	0,9984
A_{320}	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,9492	0,9989
A_{322}	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,9514	0,9995
A_{323}	0,0067	0,0067	0,0067	0,0067	0,0067	0,0067	0,0067	0,0067	0,0067	0,0067	0,9423	0,9964
A_{325}	0,0043	0,0043	0,0043	0,0043	0,0043	0,0043	0,0043	0,0043	0,0043	0,0043	0,9454	0,9977
C_{144}	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,0010	0,9423	0,9995

Thus, in terms of fuzzy sets A_t (t=1÷333) and C_k (k=1÷144), the DJIA index time series and its interpretation in PE notations are summarized in Table 5.

Table 5. Reconstruction of the DJIA index Time Series in Terms of PE FS

SN	Date	DJIA	FS	PE	Criterion	Detailing	PE FS
1	06/15/2018	25090,5	A_1	0,6062	C_{55}	$A_1 \cap A_{107} \cap A_{164}$	0,6051
2	06/18/2018	24987,5	A_2	0,5939	C_{51}	$A_2 \cap A_{93}$	0,5936
3	06/19/2018	24700,2	A_3	0,5330	C_{41}	$A_3 \cap A_{18} \cap A_{150}$	0,5335
………							
325	09/30/2019	26916,8	A_{325}	0,9977	C_{144}	$A_{264} \cap A_{265} \cap A_{269} \cap A_{275} \cap A_{279} \cap A_{311} \cap A_{312} \cap A_{315} \cap \ldots \cap A_{320} \cap A_{322} \cap A_{323} \cap A_{325}$	0,9995
326	10/01/2019	26573,0	A_{326}	0,9627	C_{134}	$A_{70} \cap A_{212} \cap A_{284} \cap A_{326} \cap A_{329}$	0,9643
327	10/02/2019	26078,6	A_{327}	0,8741	C_{111}	$A_{251} \cap A_{327}$	0,8764
328	10/03/2019	26201,0	A_{328}	0,9048	C_{119}	$A_{103} \cap A_{298} \cap A_{328}$	0,9047
329	10/04/2019	26573,7	A_{329}	0,9627	C_{134}	$A_{70} \cap A_{212} \cap A_{284} \cap A_{326} \cap A_{329}$	0,9643
330	10/07/2019	26478,0	A_{330}	0,9513	C_{131}	$A_{71} \cap A_{80} \cap A_{285} \cap A_{330} \cap A_{333}$	0,9538
331	10/08/2019	26164,0	A_{331}	0,8959	C_{117}	A_{331}	0,8959
332	10/09/2019	26346,0	A_{332}	0,9321	C_{125}	$A_{204} \cap A_{332}$	0,9320
333	10/10/2019	26496,7	A_{333}	0,9534	C_{131}	$A_{71} \cap A_{80} \cap A_{285} \cap A_{330} \cap A_{333}$	0,9538

4. DJIA Index Time Series Forecasting

Identified and divided into groups, the 1st order relationships are the fuzzy relations that reflects the cause-effect relations between qualitative (fuzzy) estimates of the DJIA index as a linguistic variable and its fuzzy predict:

- *unambiguously*, in the form of the implication: "If x_t is C_i, then x_{t+1} is C_j" (t=1 ÷ 333; i, j = 1 ÷ 144),
- *ambiguously*, in the form of the implication: "If x_t is C_i, then x_{t+1} is $C_{j(1)}$ or $C_{j(2)}$ or … or $C_{j(p)}$" (t = 1 ÷ 333; i, $j(1)$, $j(2)$,…, $j(p)$ = 1 ÷ 144).

If in an unambiguous case everything is extremely clear, then in the case of the presence of two or more alternative fuzzy conclusions, the predict is consolidated using the logical operator "OR". In particular, for fuzzy relations $C_{36} \Rightarrow C_{21}$, C_{51}, C_{52} we have: "If x_t is C_{36}, then x_{t+1} is C_{21} or C_{51} or C_{52}", where the generalized predict is the fuzzy set

$F = C_{21} \cup C_{51} \cup C_{52}$ with the membership function [8]:

$$\mu_F(u) = \mu_{C_{21} \cup C_{51} \cup C_{52}}(u) = \max\{\mu_{C_{21}}(u), \mu_{C_{51}}(u), \mu_{C_{52}}(u)\}.$$

Table 6 presents generalization of fuzzy sets representing the consequences in the 1st order relationship groups.

Table 6. Generalizing Fuzzy Sets Reflecting the Consequences in the 1st Order Groups

Fuzzy predict	Membership function values											PE
	0,0000	0,1000	0,2000	0,3000	0,4000	0,5000	0,6000	0,7000	0,8000	0,9000	1,0000	
F_1	0,9138	0,2051	0,2051	0,2051	0,2051	0,2051	0,1613	0,0913	0,0413	0,0412	0,0412	0,0767
F_2	0,5875	0,4978	0,4978	0,4978	0,4978	0,3892	0,2992	0,2292	0,1792	0,1492	0,0031	0,2758
F_3	0,9138	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0412	0,0226
F_4	0,7193	0,3392	0,3392	0,3392	0,3392	0,2896	0,1996	0,1296	0,0796	0,0496	0,0396	0,1491
F_5	0,6379	0,4414	0,4414	0,4414	0,4414	0,3475	0,2575	0,1875	0,1375	0,1075	0,0149	0,2209
F_6	0,7489	0,2985	0,2985	0,2985	0,2985	0,2730	0,1830	0,1130	0,0630	0,0330	0,0230	0,1257
F_7	0,8173	0,1960	0,1960	0,1960	0,1960	0,1960	0,1606	0,0906	0,0406	0,0106	0,0006	0,0785
………												
F_{136}	0,0117	0,0217	0,0463	0,0963	0,1161	0,1161	0,1161	0,1161	0,1161	0,1161	0,8730	0,9500
F_{137}	0,0291	0,0391	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,0684	0,9215	0,9666
F_{138}	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,0337	0,9173	0,9816
F_{139}	0,0422	0,0422	0,0585	0,0834	0,0834	0,0834	0,0834	0,0834	0,0834	0,0834	0,9800	0,9629
F_{140}	0,0337	0,0337	0,0625	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,9173	0,9648
F_{141}	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,0147	0,9423	0,9922
F_{142}	0,0369	0,0469	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,0532	0,9423	0,9730
F_{143}	0,0422	0,0422	0,0625	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,9800	0,9662
F_{144}	0,0337	0,0337	0,0625	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,0737	0,9423	0,9657

Thus, in the PE notation of fuzzy sets, the predictive model built on the basis of 1st order internal relationships, or simply, the 1st order model induces defuzzified outputs (predictions) on the scale of the interval [0, 1], which are presented in Table 7. The geometric interpretation of the model in comparison with the DJIA time series in the PE notation of fuzzy sets is shown in Fig. 4.

Table 7. Predictive Model of the DJIA Time Series

№	Date	DJIA fuzzy analogue		FS	1st order relationships group	Model output (predict)		Detailing the fuzzy output
		FS	PE			FS	PE	
1	06/15/2018	A_1	0.6062	C_{55}	$C_{55} \Rightarrow C_{51}, C_{54}, C_{59}$	–	–	
2	06/18/2018	A_2	0.5939	C_{51}	$C_{51} \Rightarrow C_{40}, C_{41}$	F_{55}	0.6203	$C_{51} \cup C_{54} \cup C_{59}$
3	06/19/2018	A_3	0.5330	C_{41}	$C_{41} \Rightarrow C_{28}, C_{39}, C_{49}$	F_{51}	0.5318	$C_{40} \cup C_{41}$
4	06/20/2018	A_4	0.5219	C_{39}	$C_{39} \Rightarrow C_{32}$	F_{41}	0.4557	$C_{28} \cup C_{39} \cup C_{49}$
5	06/21/2018	A_5	0.4723	C_{32}	$C_{32} \Rightarrow C_{23}, C_{32}, C_{36}$	F_{39}	0.4729	C_{32}
6	06/22/2018	A_6	0.5024	C_{36}	$C_{36} \Rightarrow C_{21}, C_{51}, C_{52}$	F_{32}	0.4546	$C_{23} \cup C_{32} \cup C_{36}$
7	06/25/2018	A_7	0.4220	C_{21}	$C_{21} \Rightarrow C_{23}$	F_{36}	0.5240	$C_{21} \cup C_{51} \cup C_{52}$
8	06/26/2018	A_8	0.4280	C_{23}	$C_{23} \Rightarrow C_{18}, C_{38}$	F_{21}	0.4285	C_{23}
………								
328	10/03/2019	A_{328}	0.9048	C_{119}	$C_{119} \Rightarrow C_{105}, C_{122}, C_{134}$	F_{111}	0.8872	$C_{113} \cup C_{119}$
329	10/04/2019	A_{329}	0.9627	C_{134}	$C_{134} \Rightarrow C_{111}, C_{131}, C_{132}$	F_{119}	0.8777	$C_{105} \cup C_{122} \cup C_{134}$
330	10/07/2019	A_{330}	0.9513	C_{131}	$C_{131} \Rightarrow C_{81}, C_{117}, C_{127}, C_{129}$	F_{134}	0.8944	$C_{111} \cup C_{131} \cup C_{132}$
331	10/08/2019	A_{331}	0.8959	C_{117}	$C_{117} \Rightarrow C_{125}$	F_{131}	0.8127	$C_{81} \cup C_{117} \cup C_{127} \cup C_{129}$
332	10/09/2019	A_{332}	0.9321	C_{125}	$C_{125} \Rightarrow C_{116}, C_{131}$	F_{117}	0.9320	C_{125}
333	10/10/2019	A_{333}	0.9534	C_{131}	$C_{131} \Rightarrow C_{81}, C_{117}, C_{127}, C_{129}$	F_{125}	0.9055	$C_{116} \cup C_{131}$
						MSE	0.0020	
						MAPE	4.4879	
						MPE	-0.4391	

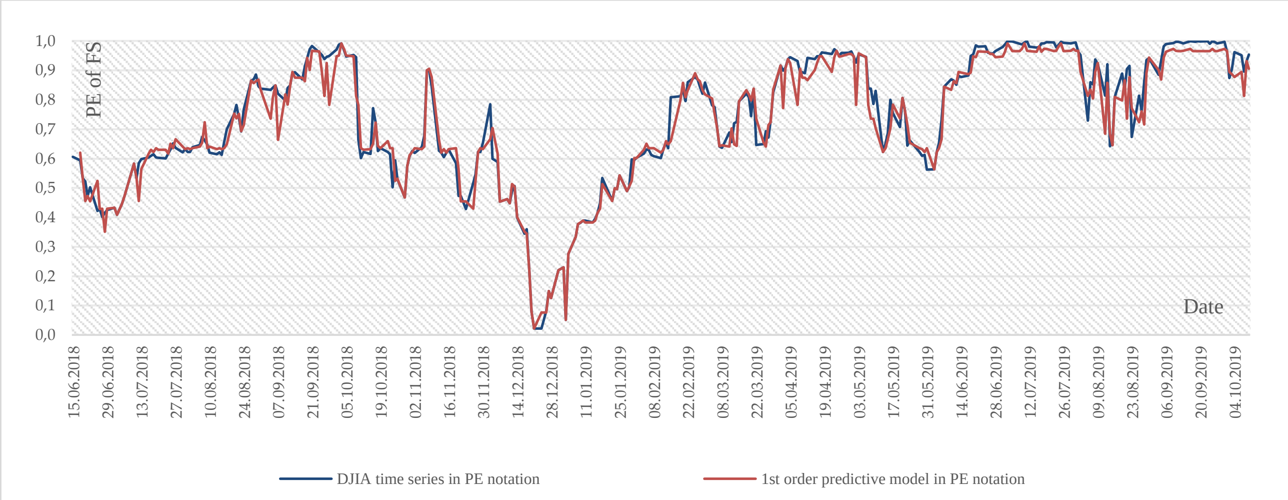

Fig. 4. 1st Order Predictive Model in PE Notation

At the end of Table 7, the values of the statistical evaluation criteria MSE, MAPE and MPE are presented, which reflect the adequacy of the proposed predictive model. Errors according to these criteria are calculated by following formulas [20]:

$$\text{MSE} = \frac{1}{m}\sum_{t=1}^{m}(F_t - A_t)^2,$$

$$\text{MAPE} = \frac{1}{m}\sum_{t=1}^{m}\frac{|F_t - A_t|}{A_t}\times 100\%,$$

$$\text{MPE} = \frac{1}{m}\sum_{t=1}^{m}\frac{F_t - A_t}{A_t}\times 100\%,$$

where m is the length of the time series; A_t is the DJIA at time t; F_t is the predict of the A_t.

The MSE criterion is most often used when choosing the optimal predictive model and highlights possible significant errors in forecasts. In our case, MSE=0.0020 (Table 7) indicates that the prediction error is too low.

The MAPE shows how large the forecast errors are compared to the actual DJIA values.

MPE is a more informative criterion for assessing the adequacy of the forecasting model, which determines the “bias” of the constructed predict, that is, its permanent underestimation or overestimation.

In our case, the MPE = -0.4391% (Table 7) reflects a slight bias of the predictive model, which does not exceed the normative 5% threshold on the left[1].

5. Forecasting the DJIA Index Time Series in Nominal Values Using the 1st Order Fuzzy Model

After forecasting the DJIA index time series in terms of point estimates of the fuzzy outputs (predicts) on the scale of a single segment, it's time to reflect them in nominal values. To do this, the three-layer neural network is used [2].

To build an approximation neural network in MATLAB notation (Fig. 5), the set of training pairs $\{(A_t^{\text{def}}, x_t)\}_{t=1}^{136}$ is chosen as a basis, where x_t is the DJIA value at time t; A_t^{def} is the defuzzified value (PE) of the fuzzy set A_t, reflecting the x_t. After training, testing and validation (Fig. 6), the neural network can be approximate the continuous function presented in tabular form (Table 8).

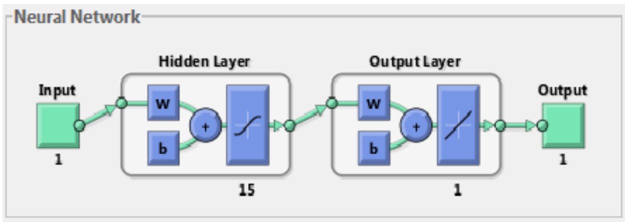

Fig. 5. Neural Network in MATLAB Notation

The trained neural network induces at its output the nominal predictive values of the DJIA, corresponding to the defuzzified analogues of fuzzy outputs presented in Table 8. The predictions obtained in this way are presented in Table 9, and the 1st order model is interpreted in Fig. 7 against the background of the original DJIA index time series.

[1] If there were a large negative MPE, then the constructed model would be considered “overestimating”. If the MPE indicator would reflect a large positive percentage value, i.e. beyond the 5% threshold on the right, then the model would be considered “underestimating”.

[2] It is clear that by a simple mapping $x=a+t(b-a)$, where $x\in[a, b]$ (in our case, $x\in[21752.2, 27359.2]$), $t\in[0, 1]$ it is impossible to reflect point estimates of fuzzy outputs, because obviously, the relationship between them is non-linear.

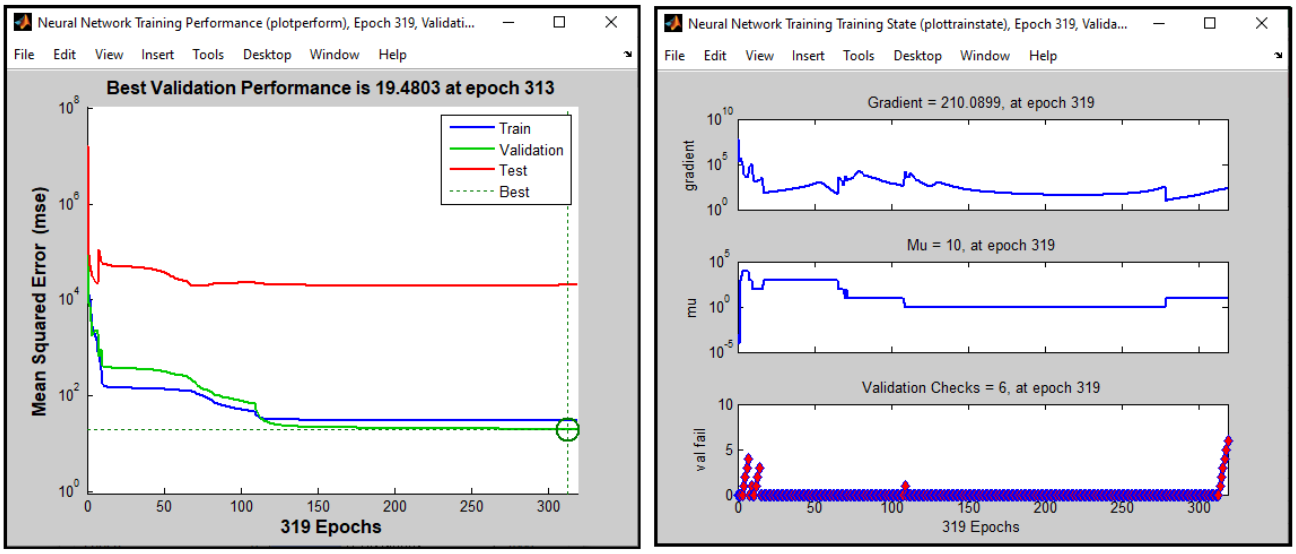

Fig. 6. Neural Approximation of the Function $x_t = f(A_t^{\text{def}})$ in MATLAB Notation

Table 8. Tabular Representation of the Function $x_t = f(A_t^{\text{def}})$

t	A_t^{def}	x_t	t	A_t^{def}	x_t	t	A_t^{def}	x_t	t	A_t^{def}	x_t
1	0,6062	25090,5	35	0,6392	25462,6	69	0,9835	26743,5	103	0,9025	26191,2
2	0,5939	24987,5	36	0,6467	25502,2	70	0,9615	26562,1	104	0,8462	25989,3
3	0,5330	24700,2	37	0,6736	25628,9	71	0,9529	26492,2	105	0,6264	25387,2
4	0,5219	24657,8	38	0,6614	25583,8	72	0,9384	26385,3	106	0,6202	25286,5
5	0,4723	24461,7	39	0,6474	25509,2	73	0,9465	26439,9	107	0,6051	25080,5
6	0,5024	24580,9	40	0,6204	25313,1	74	0,9489	26458,3	108	0,6205	25289,3
7	0,4220	24252,8	41	0,6143	25187,7	75	0,9712	26651,2	109	0,6314	25413,2
8	0,4280	24283,1	42	0,6213	25299,9	76	0,9870	26773,9	110	0,5844	25017,4
9	0,4013	24117,6	43	0,6125	25162,4	77	0,9921	26828,4	111	0,4733	24465,6
10	0,4154	24216,1	44	0,6569	25558,7	78	0,9687	26627,5	112	0,4730	24464,7
11	0,4256	24271,4	45	0,7003	25669,3	79	0,9475	26447,1	113	0,4286	24286,0
12	0,4332	24307,2	46	0,7517	25758,7	80	0,9523	26486,8	114	0,5173	24640,2
13	0,4089	24174,8	47	0,7827	25822,3	81	0,9452	26430,6	115	0,5457	24748,7
14	0,4448	24356,7	48	0,7383	25733,6	82	0,6641	25598,7	116	0,6237	25366,4
15	0,4709	24456,5	49	0,6924	25657,0	83	0,6021	25052,8	117	0,6221	25338,8
16	0,5530	24776,6	50	0,7677	25790,4	84	0,6220	25340,0	118	0,6533	25538,5
17	0,5835	24919,7	51	0,8656	26049,6	85	0,6159	25250,6	119	0,7845	25826,4
18	0,5330	24700,5	52	0,8699	26064,0	86	0,7716	25798,4	120	0,5991	25027,1
19	0,5844	24924,9	53	0,8862	26124,6	87	0,7230	25706,7	121	0,5881	24947,7
20	0,5981	25019,4	54	0,8454	25986,9	88	0,6254	25379,5	122	0,4531	24389,0
21	0,6034	25064,4	55	0,8378	25964,8	89	0,6359	25444,3	123	0,4622	24423,3
22	0,6090	25119,9	56	0,8337	25952,5	90	0,6225	25317,4	124	0,4482	24370,2
23	0,6150	25199,3	57	0,8412	25975,0	91	0,6145	25191,4	125	0,4892	24527,3
24	0,6034	25064,5	58	0,8485	25995,9	92	0,5030	24583,4	126	0,5065	24597,4
25	0,6027	25058,1	59	0,8211	25916,5	93	0,5936	24984,6	127	0,3992	24100,5
26	0,6011	25044,3	60	0,7978	25857,1	94	0,5298	24688,3	128	0,3448	23593,0
27	0,6173	25241,9	61	0,8398	25971,1	95	0,4673	24442,9	129	0,3600	23675,6
28	0,6316	25414,1	62	0,8495	25998,9	96	0,5752	24874,6	130	0,2192	23323,7
29	0,6513	25527,1	63	0,8914	26146,0	97	0,6086	25115,8	131	0,0785	22859,6
30	0,6371	25451,1	64	0,8936	26154,7	98	0,6255	25380,7	132	0,0226	22445,4
31	0,6218	25306,8	65	0,8693	26062,1	99	0,6189	25270,8	133	0,0216	21792,2
32	0,6318	25415,2	66	0,9147	26247,0	100	0,6391	25461,7	134	0,0821	22878,5
33	0,6223	25333,8	67	0,9416	26405,8	101	0,6778	25635,0	135	0,1491	23138,8
34	0,6227	25326,2	68	0,9720	26657,0	102	0,8999	26180,3	136	0,1257	23062,4

Table 9. DJIA Time Series Forecasting Taking into Account 1st Order Internal Relationships

SN	Date	DJIA	Predict
1	06/15/2018	25090,5	
2	06/18/2018	24987,5	25299
3	06/19/2018	24700,2	24694
4	06/20/2018	24657,8	24400
5	06/21/2018	24461,7	24463
6	06/22/2018	24580,9	24396
7	06/25/2018	24252,8	24664
8	06/26/2018	24283,1	24285
9	06/27/2018	24117,6	24291
10	06/28/2018	24216,1	23537
11	06/29/2018	24271,4	24292
……………………………………………………………………………			
322	09/25/2019	26970,7	26661
323	09/26/2019	26891,1	26598
324	09/27/2019	26820,3	26598
325	09/30/2019	26916,8	26661
326	10/01/2019	26573,0	26598
327	10/02/2019	26078,6	26157
328	10/03/2019	26201,0	26129
329	10/04/2019	26573,7	26093
330	10/07/2019	26478,0	26157
331	10/08/2019	26164,0	25895
332	10/09/2019	26346,0	26342
333	10/10/2019	26496,7	26204
		MSE	**72100,5**
		MAPE	**0,6830**
		MPE	**-0,2530**

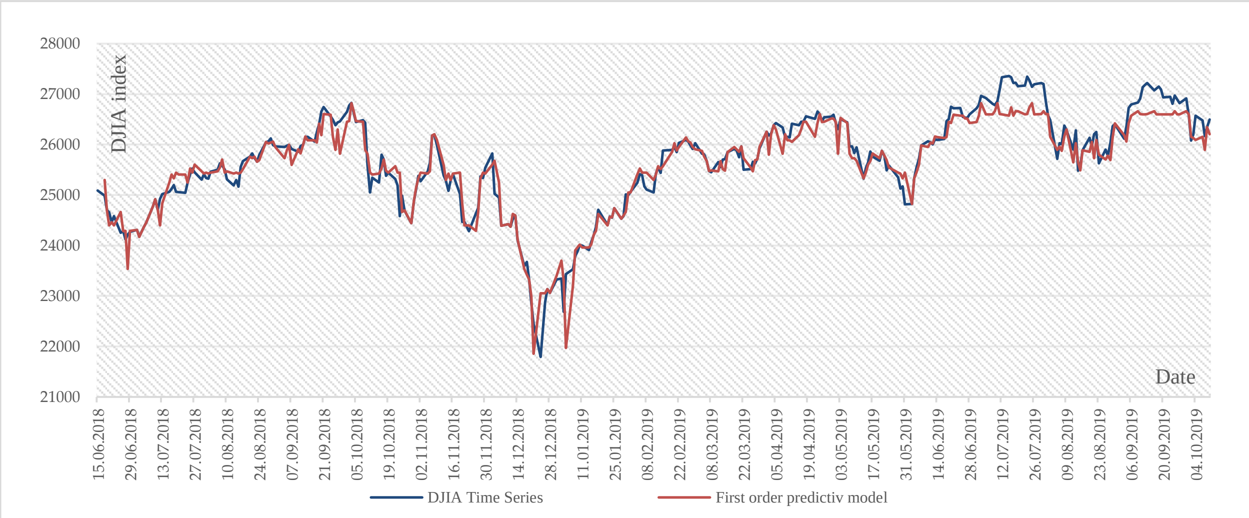

Fig. 7. 1st Order Predictive Model at Nominal Values of DJIA Index

6. Discussion of Final Results

At the end of Table 9, the values of indicators MSE = 72100.5, MAPE = 0.6830 and MPE = -0.2530 are presented. In this case, the MSE value reflects a relatively large prediction error, which is explained by insufficiently satisfactory training of the neural network ($\varepsilon = 19.4803$, Fig. 4). Nevertheless, the MAPE indicator demonstrates an acceptable forecast error in comparison with the actual values of the DJIA time series. MPE, as a more informative criterion, reflects a slight bias of the prognostic model, which does not exceed the normative 5% threshold on the left.

Comparing the two proposed models, it is easy to see that the predictive model in nominal values of the DJIA index is significantly inferior to the predictive model of the DJIA index in PE terms of the corresponding fuzzy sets. Therefore, applying the predictive model in PE terms, as the predict of the DJIA index for the 334th day, the number $A^{def}_{334} = 0.8127$ is obtained (PE of the fuzzy predict $F_{131} = C_{81} \cup C_{117} \cup C_{127} \cup C_{129}$), which is interpreted in nominal value as 26204 using the neural network (Fig. 5).

7. Conclusion

On the example of the volatile DJIA index time series, the fuzzy inference based method of data fuzzification was proposed. Due to the redundancy of fuzzy interpretations of weakly structured data compiled according to the number of the DJIA index, clustering of fuzzy sets was carried out and appropriate set of corresponding evaluation criteria was formed. The application of this approach to data fuzzification made it possible to recreate the fuzzy analogue of the weakly structured time series and, on its basis, apply one or another predictive model.

The article considered the predictive model based on internal relationships of the 1st order. Of course, one should also consider internal relationships of the 2nd and higher orders, which would provide additional resources for a qualitative improvement of the proposed approach. In this case, the results obtained could be compared with existing approaches, for example, with the predictive models of Q. Song and B. Chissom, N. Kumar et al., S. Chen, C. Cheng, J. Poulsen. But on account of the cumbersome calculations this was not possible.

References

1. Song, Q; Chissom, B.S. (1993). Forecasting enrollments with fuzzy time series – part I. Fuzzy Sets and Systems, 54, 269-277. DOI: 10.1016/0165-0114(93)90355-L
2. Song, Q; Chissom, B.S. (1994). Forecasting enrollments with fuzzy time series – part II. Fuzzy Sets and Systems, 62, 1-8. DOI: 10.1016/0165-0114(94)90067-1
3. Kumar, N; et al. (2010). Fuzzy time series forecasting of wheat production. International Journal on Computer Science and Engineering, 2(3), 635-640.
4. Chen, S.M. (1996). Forecasting enrollments based on fuzzy time series. Fuzzy Sets and Systems, 81, 311-319. DOI: 10.1016/0165-0114(95)00220-0
5. Chen, S.M. (2002). Forecasting enrollments based on high-order fuzzy time series. Cybernetics and Systems, 33(1), 1-16. DOI: 10.1080/019697202753306479
6. Chen, S.M; Wang, N.Y. (2010). Fuzzy forecasting based on fuzzy-trend logical relationship groups. IEEE Trans. Systems, Management, and Cybernetics, Part B: Cybernetics, 40(5), 1343-1358. DOI: 10.1109/TSMCB.2009.2038358
7. Chen, S.M; Hwang, J.R. (2000). Temperature prediction using fuzzy time series. IEEE Trans. Systems, Management, and Cybernetics, Part B: Cybernetics, 30 (2), 263-275. DOI: 10.1109/3477.836375
8. Chen, S.M; Chung, N.Y. (2006). Forecasting enrollments of students by using fuzzy time series and genetic algorithms. International Journal of Information and Management Sciences, 17(3), 1.-17. DOI: 10.1002/0470024569.ch1
9. Chen, S.M; Chung, N.Y. (2006). Forecasting enrollments using highorder fuzzy time series and genetic algorithms. International Journal of Information and Management Sciences, 21, 485-501. DOI: 10.1002/int.20145
10. Chen, S.M; Kao, P.Y. (2013). TAIEX forecasting based on fuzzy time series, particle swarm optimization techniques and support vector machines. Information Sciences, 247, 62-71. DOI: 10.1016/j.ins.2013.06.005
11. Cheng, C.H; Chang, J.R; Yeh, C.A. (2006). Entropy-based and trapezoid fuzzification fuzzy time series approaches for forecasting IT project cost. Technological Forecasting & Social Change, 73, 524–542. DOI: 10.1016/j.techfore.2005.07.004
12. Poulsen, J.R. (2009). Fuzzy time series forecasting – developing a new forecasting model based on high order fuzzy time series. AAUE: CIS 4.
13. Huarng, K. (2001). Heuristic models of fuzzy time series for forecasting. Fuzzy Sets and Systems, 123(3), 369-386. DOI: 10.1016/S0165-0114(00)00093-2
14. Huang, Y.L; Horng, S.J; He, M; Fan, P; Kao, T.W; Khan, M.K; Kuo, I.H. (2011). A hybrid forecasting model for enrollments based on aggregated fuzzy time series and particle swarm optimization. Expert Systems with Applications, 38(7), 8014-8023. DOI: 10.25046/aj0601147
15. Uslu, V.R; Bas, E; Yolcu, U; Egrioglu, E. (2014). A fuzzy time series approach based on weights determined by the number of recurrences of fuzzy relations. Swarm and Evolutionary Computation, 15, 19-26. DOI: 10.5923/j.ajis.20150503.01
16. Mardanov, M.J; Rzayev, R.R; Alizada, P.E. (2021). About one approach to data fuzzification on the example of the Dow Jones index time series. Mathematical Machines and Systems, 1, 108-115 (in Russian). DOI: 10.34121/1028-9763-2020-2-3-13
17. Dow Jones industrial index. https://www.ru.tradingview.com/symbols/DJ-DJI/
18. Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning. Information Sciences, 8(3), 199-249. DOI: 10.1016/0020-0255(75)90036-5
19. Ortiz-Arroyo, D; Poulsen, J.R. (2018). A weighted fuzzy time series forecasting model. Indian Journal of Science and Technology, 11(27), 1-11. DOI: 10.17485/ijst/2018/v11i27/130708
20. Lewis, K.D. (1986). Methods for forecasting economic indicators. Moscow: Finance and Statistics (in Russian).