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Uncertainty Structure of Parameterised Finite Groups

J.Saranya¹, Dr. V. Ramadoss²

¹Research Scholar, Department of Mathematics, PRIST University, Tanjore, Tamilnadu

²Professor, Department of Mathematics, PRIST University, Tanjore, Tamilnadu

Abstract: In this paper, we study soft normal subgroups of subgroups, direct product of fuzzy soft normal subgroups and their properties.

Keywords: Fuzzy set, fuzzy relation, soft set, s-norm, normal subgroup, similar.

I. INTRODUCTION

In various algebra, a normal subdivision group is a subgroup that is invariant under opposition by members of the group of which it is a part. Alternatively a subgroup H of a group G is normal in G if and only if $eH = He$ for all e in G [4]. For centuries uncertain theory[5] and error study have been the only models to treat imprecision and uncertainty in [3]. Even though [2] recently a lot of new models have been analysed for handling incomplete information. In this article, we obey the direct product form of uncertainty function.

II. PRELIMINARIES

A. Definition 2.1:

A uncertainty subset of G , we mean a function $cv: G \rightarrow I$ The set of all uncertainty subsets of G is known the I-power set of G and is denoted by I^G . A uncertainty combination, on G we mean a map $cv: G \times G \rightarrow I$ Denote by $F_R(G)$, the set of all uncertainty relations on G .

B. Definition 2.2:

Let $cv_1, cv_2 \in F_R(G)$ and $x, y \in G$. we set

- (i) $cv_1 \subseteq cv_2$ if and only if $cv_1(x, y) \leq cv_2(x, y)$
- (ii) $cv_1 = Acv_2$ if and only if $cv_1(x, y) = cv_2(x, y)$.

C. A Co-norm S is a map $cv: I \times I \rightarrow I$ having the following Rules:

- (cv₁) $cv(xm^* + p, 0) = x$ (neutral element)
- (cv₂) $cv(xm^* + p, ym^* + c) \leq cv(x, z)$ if $y \leq z$ (monotonicity)
- (cv₃) $cv(xm^* + p, ym^* + c) = cv(y, x)$ (commutativity)
- (cv₄) $cv(x, cv(ym^* + p, z)) = cv(cv(x, ym^* + c), z)$ (associativity) for all $x, y, z \in I$

D. Definition 2.4:

Let 'j' be a uncertainty parameterised subset of a group G , then 'j' is called a uncertainty parameterised subgroup of G under a co norm (S- uncertainty parameterised subgroup) if and only if for all $x, y \in G$.

- (i) $cv(xym^* + c) \leq cv(j(x), j(y))$
- (ii) $cv(x^{-1}m^* + c) \leq cv(x)$.

Denote by $cv(G)$, the set of all co norm- uncertainty parameterised subgroup of G .

Example 2.5: Let $G = \{1, i, -1, -i\}$ be a group with respect to . Define uncertainty subset $cv: G \rightarrow [I]$ as

$$cv(x) = \begin{cases} am^* / b, & \text{if } x = 1 \\ bm^* / c, & \text{if } x = -1 \\ am^* / c, & \text{if } x = \pm i \end{cases}$$

E. Definition 2.5:

Let $f : \frac{G_1}{H_1} \rightarrow \frac{G_2}{H_2}$, $cv1 \in [I]_{H_1}^{G_1}$ and $cv2 \in [I]_{H_2}^{G_2}$

Define $f(cv1) \in [I]_{H_2}^{G_2}$ as $f(e_1 H_1) = e_2 H_2$ if $f^{-1}(e_2 H_2) \neq \emptyset$

$$f(cv1)(e_2 H_2) = \begin{cases} \inf \{cv1(e_1 H_1) / e_1 H_1 \in \frac{G_1}{H_1} \\ 0, \text{ if } f^{-1}(e_2 H_2) = \emptyset \end{cases}$$

for all $e_2 H_2 \in \frac{G_1}{H_1}$.

F. Definition 2.6:

A uncertainty parameterised relation $cv : Group \times Group \rightarrow [I]$ on a group G is a S- uncertainty parameterised combinations on G if the following conditions are satisfied.

- (i) $cv(xm^* + p, x) = 0$
- (ii) $cv(xn^* + p, y) = cv(y, x)$
- (iii) $cv(xm^* + c, z) \leq cv(cv(x, y), cv(y, z))$, for all $x, y, z \in G$.

G. Example 2.7:

Let $Group = (Z, +)$ be a group of integer numbers. Set $cv : Z^* \times Z^* \rightarrow [I]$ by $cv(x, y) = \begin{cases} 0, \text{ if } x = y \\ dm^* / c, \text{ otherwise} \end{cases}$

H. Definition 2.8

Let 'G' be a group and 'H' be a normal subgroup of G . Then $cv_{\frac{G}{H}} : \frac{G}{H} \rightarrow [I]$ can be viewed by

$$cv_{\frac{G}{H}}(xm^* + cH) = \Delta(xm^* + c, h), \text{ for all } x \in G \text{ and } h \in H.$$

III. STRUCTURES OF VARIOUS CHARACTERISATIONS

A. Proposition 3.1:

Let $j_H \in cv(HM^*)$ and cv be similar co-norm. Then $j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right)$.

Proof: Let $xH, yH \in \frac{G}{H}$ and $j_H \in cv(HM)$.

Then, $j_{\frac{G}{H}}(xm^* + cHym^* + cH) = j_{\frac{G}{H}}(xym + cH) = \Delta(xym + c, h)$

$$\begin{aligned} &= cv(j_H(xym + c), j_H(h)) \\ &\leq cv(cv(j_H(x), j_H(y)), j_H(h)) \\ &= cv(cv(j_H(xm + c), j_H(ym + b)), cv(j_H(h), j_H(h))) \\ &= cv(cv(j_H(xm + p), j_H(h)), cv(j_H(ym^* + c), j_H(h))) \\ &= cv(\Delta(x, h), \Delta(y, h)) \\ &= cv\left(j_{\frac{G}{H}}(xHm + c), j_{\frac{G}{H}}(yHm + c)\right) \end{aligned}$$

Also, $j_{\frac{G}{H}}(xH)^{-1} = cv_{\frac{G}{H}}(x^{-1}m + cH) = \Delta(x^{-1}, h)$

$$\begin{aligned}
 &= cv(j_H(x^{-1}m + c), j_H(h)) = cv(j_H(xm + p), j_H(h)) \\
 &= w(x, h) = A_{\frac{G}{H}}(xHm + c). \text{Therefore, } j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right).
 \end{aligned}$$

B. Proposition 3.2:

If cv be similar co-norm, then for all $xH \in \frac{G}{H}$, and $n \geq 1$,

$$\begin{aligned}
 \text{(i)} \quad & j_{\frac{G}{H}}(H) \leq j_{\frac{G}{H}}(xHm + c) \\
 \text{(ii)} \quad & j_{\frac{G}{H}}(xHm + c)^n \leq j_{\frac{G}{H}}(xHm * + c) \\
 \text{(iii)} \quad & j_{\frac{G}{H}}(xHm * + c) = j_{\frac{G}{H}}(xHm + c)^{-1}
 \end{aligned}$$

Proof: Let $xH \in \frac{G}{H}$, and $n \geq 1$,

From Proposition 3.1, we have that $j_{\frac{G}{H}} \in cv\left(\frac{G}{H}\right)$

$$\begin{aligned}
 \text{(i)} \quad & j_{\frac{G}{H}}(H) = j_{\frac{G}{H}}(xx^{-1}Hm * + cp) \\
 & = j_{\frac{G}{H}}(xHx^{-1}Hm + p) \\
 & \leq cv\left(j_{\frac{G}{H}}(xHm * + p), j_{\frac{G}{H}}(x^{-1}Hm + p)\right) \\
 & \leq cv\left(j_{\frac{G}{H}}(xHm + c), j_{\frac{G}{H}}(xHm + c)\right) \\
 & = j_{\frac{G}{H}}(xHm * + c) \\
 \text{(ii)} \quad & j_{\frac{G}{H}}(xHm * + c)^n = j_{\frac{G}{H}}m * + p(xHm * + p, xHm * + p, xHm * + p, xHm * + p, \dots, n \text{ times}) \\
 & \leq cv\left(j_{\frac{G}{H}}(xHm * + p), j_{\frac{G}{H}}(xHm * + p), j_{\frac{G}{H}}(xHm * + p), \dots, n \text{ times}\right) \\
 & = j_{\frac{G}{H}}(xHm * + c) \\
 \text{(iii)} \quad & j_{\frac{G}{H}}(xHm * + c) = j_{\frac{G}{H}}(x^{-1}H) \\
 & \leq j_{\frac{G}{H}}(x^{-1}H) \\
 & \leq j_{\frac{G}{H}}(xHm + c) \text{ . So, } j_{\frac{G}{H}}(xH) = j_{\frac{G}{H}}(x^{-1}Hm * + c)
 \end{aligned}$$

C. Proposition 3.3:

Let $j_{\frac{G}{H}}$ be a uncertainty parameterised set of a finite group $\frac{G}{H}$ and 'cv' be similar co-norm. If $j_{\frac{G}{H}}$ satisfies 2.6, then

$$j_{\frac{G}{H}} \in cvF\left(\frac{G}{H}\right).$$

Proof: Let $xHm^* + c \in \frac{G}{H}$, $x \notin H$.

Since, $\frac{G}{H}$ is finite, xH has finite number, say $n > 1$.

So, $(xHm^* + c)^n = H$ and $x^{-1}H = x^{n-1}H$.

Now by using (i) occurrence same, we have that

$$\begin{aligned} j_{\frac{G}{H}}(x^{-1}Hm + c) &= j_{\frac{G}{H}}(x^{n-1}H) = j_{\frac{G}{H}}(x^{n-2}xH) \\ &\leq cv\left(j_{\frac{G}{H}}(x^{n-2}Hm), j_{\frac{G}{H}}(xHm + c)\right) \\ &\leq cv\left(j_{\frac{G}{H}}(xHm^* + c), j_{\frac{G}{H}}(xHm^* + c), j_{\frac{G}{H}}(xHm^* + c) \dots n \text{ times}\right) \\ &= j_{\frac{G}{H}}(xHm^* + c). \end{aligned}$$

IV. CONCLUSION

Main part of this uncertainty has been discussed with its application

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