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Observations on the Hyperbola, y2 = 14x2 + 16t, t >=0 $y^2 = 14x^2 + 16^t, t \geq 0$

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Abstract: The binary quadratic equation $y^2 = 14x^2 + 16^t$ representing hyperbola is considered for finding its integer solutions. A few interesting properties among the solutions are presented. Also, we present infinitely many positive integer solutions in terms of Generalized Fibonacci sequences of numbers, Generalized Lucas sequences of numbers.
Keywords: Binary Quadratic Integral Solutions, Generalized Fibonacci Sequences of Numbers, Generalized Lucas Sequences of Numbers, Integral Solutions.

AMS Mathematics Subject Classification:11D09

Notations

$GF_n(k,s)$: Generalized Fibonacci Sequences of rank n.

$GL_n(k,s)$: Generalized Lucas Sequences of rank n.

$$t_{m,n} = n\left(I + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^m = \frac{[n(n+1)((m-2)(n+(5-m))]}{6}$$

$$\Pr_n = n(n+1)$$

$$Ct_{m,n} = \frac{mn(n+1)}{2} + 1$$

$$S_n = 6n(n-1) + 1$$

I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D non-square positive integer has been studied by various mathematics for its non-trivial integer solutions.

when D takes different integral values $[1,2,4]$. In $[3]$ infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the $y^2 = 14x^2 + 1$. In [5] a special Pythagorean triangles is obtained by employing the integral solutions of $y^2 = 182x^2 + 14^t$.

In [6] different patterns of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of $y^2 = 14x^2 + 4$. In this context one may also refer [7,8]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation $y^2 = 14x^2 + 16^t$ representing a hyperbola. A few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration a few patterns of Pythagorean triangles are obtained.

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II. METHOD OF ANALYSIS

Consider the binary quadratic equation,

$$y^2 = 14x^2 + 16^t, t \geq 0 \qquad (1)$$

with least positive integer solutions of (1) ,

$$x_0 = 4(4^t), y_0 = 15(4^t), D = 14$$

To obtain the other solutions of (1),
Consider the pellian equation is

$$y^2 = 14x^2 + 1 \qquad (2)$$

The initial solution of pellian equation is

$$\tilde{x}_0 = 4(4^t), \tilde{y}_0 = 15(4^t)$$

The general solution (x_n, y_n) of (2) is given by,

$$\tilde{x}_n = \frac{1}{2\sqrt{14}} g_n \ , \ \tilde{y}_n = \frac{1}{2} f_n$$

where,

$$f_n = (15 + 4\sqrt{14})^{n+1} + (15 - 4\sqrt{14})^{n+1}$$

$$g_n = (15 + 4\sqrt{14})^{n+1} - (15 - 4\sqrt{14})^{n+1}$$

Applying Brahmagupta lemma between (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$ the general solutions of equation (1) are found to be,

$$x_{n+1} = 2.4^t f_n + \frac{15}{2\sqrt{14}} 4^t g_n \qquad (3)$$

$$y_{n+1} = \frac{15}{2}.4^t f_n + 2\sqrt{14} 4^t g_n \qquad (4)$$

Thus (3) and (4) represent the non-zero distinct integer solutions of (1)
The recurrence relation satisfied by the solution x and y are given by

$$x_{n+3} - 30x_{n+2} + x_{n+1} = 0$$

$$y_{n+3} - 30y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x and y satisfying (1) are given in the Table 1 below,

Table 1: Numerical Examples

N	x_n	y_n
0	$4(4^t)$	$15(4^t)$
1	$3596(4^t)$	$13455(4^t)$
2	$107760(4^t)$	$403201(4^t)$
3	$3229204(4^t)$	$12082575(4^t)$
4	$96230160(4^t)$	$36006479(4^t)$

A. A Few Interesting Relations Among The Solutions Are Given Below

1) $8x_{n+1}-240x_{n+2}+8x_{n+3}=0$
2) $1118x_{n+2}-33750x_{n+1}+8y_{n+1}=0$
3) $399842x_{n+1}+13358x_{n+2}+8y_{n+2}=0$
4) $30x_{n+1}-898x_{n+2}+8y_{n+3}=0$
5) $240y_{n+1}-2x_{n+3}-898x_{n+1}=0$
6) $30x_{n+1}-30x_{n+3}+240y_{n+2}=0$
7) $2x_{n+1}-898x_{n+3}+240y_{n+3}=0$
8) $30x_{n+2}-2x_{n+1}-8y_{n+2}=0$
9) $30y_{n+3}-898y_{n+2}-112x_{n+1}=0$
10) $1399080x_{n+1}-374882y_{n+1}+2y_{n+3}=0$
11) $30x_{n+2}-2x_{n+3}+8y_{n+2}=0$
12) $2x_{n+2}-30x_{n+3}+8y_{n+3}=0$
13) $30x_{n+3}-898x_{n+2}-8y_{n+1}=0$
14) $30y_{n+3}-2y_{n+1}-112x_{n+2}=0$
15) $30y_{n+3}-30y_{n+1}-3360x_{n+2}=0$
16) $2y_{n+3}-30y_{n+2}-112x_{n+2}=0$
17) $898y_{n+2}-30y_{n+1}-112x_{n+3}=0$
18) $898y_{n+3}-2y_{n+1}-3360x_{n+3}=0$
19) $448y_{n+1}-13440y_{n+2}+448y_{n+3}=0$
20) $112x_{n+3}+30y_{n+1}-898y_{n+2}=0$

B. Each Of The Following Expression Is A Nasty Number

1) $\frac{1}{4.4^t}[180x_{2n+3}-5388x_{2n+2}+48.4^t]$
2) $\frac{1}{120.4^t}[180x_{2n+4}-161460x_{2n+2}+1440.4^t]$
3) $\frac{1}{4^t}[180y_{2n+2}-672x_{2n+2}+12.4^t]$
4) $\frac{1}{15.4^t}[180y_{2n+3}-20160x_{2n+2}+180.4^t]$
5) $\frac{1}{449.4^t}\left[3180y_{2n+4}-604128x_{2n+2}+3592.4^t\right]$
6) $\frac{1}{4.4^t}\left[3592x_{2n+4}-161460x_{2n+3}+48.4^t\right]$

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7) $\frac{1}{15.4^t}\left[3592y_{2n+2}-672x_{2n+3}+180.4^t\right]$

8) $\frac{1}{4^t}\left[3592y_{2n+3}-20160x_{2n+3}+12.4^t\right]$

9) $\frac{1}{15.4^t}\left[3592y_{2n+4}-604128x_{2n+3}+180.4^t\right]$

10) $\frac{1}{449.4^t}\left[161460y_{2n+2}-672x_{2n+4}+3592.4^t\right]$

11) $\frac{1}{15.4^t}\left[161460y_{2n+3}-20160x_{2n+4}+180.4^t\right]$

12) $\frac{1}{4^t}\left[161460y_{2n+4}-604128x_{2n+4}+12.4^t\right]$

13) $\frac{1}{4.4^t}\left[1440y_{2n+2}-48x_{2n+3}+48.4^t\right]$

14) $\frac{1}{120.4^t}\left[43152y_{2n+2}-48y_{2n+4}+1440.4^t\right]$

15) $\frac{1}{4.4^t}\left[43152y_{2n+3}-1440y_{2n+4}+48.4^t\right]$

C. Each Of The Following Expressions Is A Cubical Integer

1) $\frac{1}{4.4^t}\left[898x_{3n+3}-30x_{3n+4}+2694x_{n+1}-90x_{n+2}\right]$

2) $\frac{1}{120.4^t}\left[30x_{3n+5}-26910x_{3n+3}+90x_{n+3}-80730x_{n+1}\right]$

3) $\frac{1}{4^t}\left[30y_{3n+3}-112x_{3n+3}+90y_{n+1}-336x_{n+1}\right]$

4) $\frac{1}{15.4^t}\left[30y_{3n+4}-3360x_{3n+3}+90y_{n+2}-10080x_{n+1}\right]$

5) $\frac{1}{449.4^t}\left[30y_{3n+5}-100688x_{3n+3}+90y_{n+3}-302064x_{n+1}\right]$

6) $\frac{1}{4.4^t}\left[898x_{3n+5}-26910x_{3n+4}+2694x_{n+3}-80730x_{n+2}\right]$

7) $\frac{1}{15.4^t}\left[898y_{3n+3}-112x_{3n+4}+2694y_{n+1}-36x_{n+2}\right]$

8) $\frac{1}{4^t}\left[898y_{3n+4}-3360x_{3n+4}+2694y_{n+2}-10080x_{n+2}\right]$

9) $\frac{1}{15.4^t}\left[898y_{3n+5}-100688x_{3n+4}+2694y_{n+3}-302064x_{n+2}\right]$

10) $\frac{1}{449.4^t}\left[26910y_{3n+}-112x_{3n+5}+80730y_{n+1}-336x_{n+3}\right]$

11) $\frac{1}{15.4^t}\left[26910y_{3n+4} - 3360x_{3n+5} + 80730y_{n+2} - 10080x_{n+3}\right]$

12) $\frac{1}{4^t}\left[26910y_{3n+5} - 100688x_{3n+5} + 80730y_{n+3} - 302064x_{n+3}\right]$

13) $\frac{1}{4.4^t}\left[240y_{3n+3\backslash} - 8y_{3n+4} + 720y_{n+1} - 24y_{n+2}\right]$

14) $\frac{1}{120.4^t}\left[7192y_{3n+3\backslash} - 8y_{2n+5} + 21576y_{n+1} - 24y_{n+3}\right]$

15) $\frac{1}{4.4^t}\left[7192y_{3n+4\backslash} - 240y_{3n+5} + 21576y_{n+2} - 720y_{n+3}\right]$

D. Each Of The Following Expressions Is A Biquadratic Integer

1) $\frac{1}{4.4^t}[30x_{4n+5} - 898x_{4n+4} + 120x_{2n+3} - 3592x_{2n+2} + 24.4^t]$

2) $\frac{1}{120.4^t}[30x_{4n+6} - 26910x_{4n+4} + 120x_{2n+4} - 107640x_{2n+2} + 720.4^t]$

3) $\frac{1}{15.4^t}\left[30y_{4n+5} - 3360x_{4n+4} + 120y_{2n+3} - 13440x_{2n+2} + 90.4^t\right]$

4) $\frac{1}{4^t}\left[30y_{4n+4} - 112x_{4n+4} + 120y_{2n+2} - 448x_{2n+2} + 6.4^t\right]$

5) $\frac{1}{449.4^t}\left[30y_{4n+6} - 100688x_{4n+4} + 120y_{2n+4} - 402752x_{2n+2} + 2694.4^t\right]$

6) $\frac{1}{4.4^t}\left[898x_{4n+6} - 26910x_{4n+5} + 3592x_{2n+4} - 107640x_{2n+3} + 24.4^t\right]$

7) $\frac{1}{15.4^t}\left[898y_{4n+4} - 112x_{4n+5} + 3592y_{2n+2} + 448x_{2n+3} + 90.4^t\right]$

8) $\frac{1}{4^t}\left[898y_{4n+5} - 3360x_{4n+5} + 3592y_{2n+3} - 13440x_{2n+3} + 6.4^t\right]$

9) $\frac{1}{15.4^t}\left[898y_{4n+6} - 100688x_{4n+5} + 3592y_{2n+4} - 402752x_{2n+3} + 90.4^t\right]$

10) $\frac{1}{449.4^t}\left[26910y_{4n+4} - 112x_{4n+6} + 107640y_{2n+2} - 448x_{2n+4} + 2694.4^t\right]$

11) $\frac{1}{15.4^t}\left[26910y_{4n+5} - 3360x_{4n+6} + 107640y_{2n+4} - 402752x_{2n+4} + 90.4^t\right]$

12) $\frac{1}{4.4^t}\left[240y_{4n+4} - 8y_{4n+5} + 960y_{2n+2} - 32y_{2n+3} + 24.4^t\right]$

13) $\frac{1}{120.4^t}\left[7192y_{4n+4} - 8y_{4n+6} + 28768y_{2n+2} - 32y_{2n+4} + 720.4^t\right]$

14) $\frac{1}{4.4^t}\left[7192y_{4n+5} - 240y_{4n+6} + 28768y_{2n+3} - 960y_{2n+4} + 24.4^t\right]$

E. Each Of The Following Expression Is A Quintic Integer

1) $\frac{1}{4.4^t}\left[30x_{5n+6}-898x_{5n+5}-4490x_{3n+3}+150x_{n+4}-8980x_{n+1}+300x_{n+2}\right]$

2) $\frac{1}{120.4^t}\left[30x_{5n+7}-26910x_{5n+5}+150x_{3n=5}-134550x_{3n+3}+300x_{n+3}-269100x_{n+1}\right]$

3) $\frac{1}{4^t}\left[30y_{5n+5}-112x_{5n+5}-560_{3n+3}+150y_{3n+2}-1120x_{n+1}+300y_{n+1}\right]$

4) $\frac{1}{15.4^t}\left[480y_{n+1}-3360x_{5n+5}+150y_{3n+4}-16800x_{3n+3}-33600x_{n+1}-150y_{n+2}\right]$

5) $\frac{1}{449.4^t}\left[30y_{5n+7}-100688x_{5n+5}+150y_{3+4}-503440x_{3n+3}+300y_{n+3}-2013760x_{n+1`}\right]$

6) $\frac{1}{15.4^t}\left[898y_{5n+5}-112x_{5n+6}+4490y_{3n+3}-560x_{3n+4}+8980y_{n+1}-1120x_{n+2}\right]$

7) $\frac{1}{4.4^t}\left[898x_{5n+7}-26910x_{5n+6}+4490x_{3n+5}-134550x_{3n+4}+8980x_{n+3}-269100x_{n+2}\right]$

8) $\frac{1}{4^t}\left[898y_{5n+6}-3360x_{5n+6}+4490y_{3n+4}-16800x_{3n+4}+8980y_{n+2}-33600x_{n+2}\right]$

9) $\frac{1}{15.4^t}\left[898y_{5n+7}-100688x_{5n+6}+4490y_{3n+5}-503440x_{3n+4}+8980y_{n+3}-2013760x_{n+2}\right]$

10) $\frac{1}{449.4^t}\left[26910y_{5n+5}-112x_{5n+7}+134550y_{3n+3}-560x_{3n+5}+269100y_{n+1}-2240x_{n+3}\right]$

11) $\frac{1}{15.4^t}\left[26910y_{5n+6}-3360x_{5n+7}+134550y_{n+4}-16800x_{3n+5}+269100y_{n+2}-33600x_{n+3}\right]$

12) $\frac{1}{4^t}\left[26910y_{5n+7}-100688x_{5n+7}+134550y_{n+5}-503440x_{3n+5}+269100y_{n+3}-2013760x_{n+3}\right]$

13) $\frac{1}{4.4^t}\left[240y_{5n+5}-8y_{5n+6}+1200y_{3n+3}-40y_{3n+4}-1320y_{n+2}+3640y_{n+1}\right]$

14) $\frac{1}{120.4^t}\left[7192y_{5n+5}-8y_{5n+7}+3596y_{3n+3}-40y_{3n+4}+104284y_{n+1}-80y_{n+3}\right]$

15) $\frac{1}{4.4^t}\left[7192y_{5n+6}-240y_{5n+7}+35960y_{3n+4}-1200y_{3n+5}+71920y_{n+2}-2400y_{n+3}\right]$

16) The solution of (1) in terms of special integers namely, generalized Fibonaci GF_n and Lucas GL_n are exhibited below,

$$x_{n+1}=2(4^t)GL_{n+1}(30,-1)+60(4^t)GF_{n+1}(30,-1)$$

$$y_{n+1}=\frac{15}{2}(4^t)GL_{n+1}(30,-1)+224GF_{n+1}(30,-1)$$

III. REMARKABLE OBSERVATION

1) Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in table 2 below

Table 2: Hyperbolas

S.NO	Hyperbola	(X,Y)
1	$X^2-224Y^2=64(4^t)^2$	$(898\sqrt{14}x_{n+1}-30\sqrt{14}x_{n+2},\ 60x_{n+1}-2x_{n+2})$
2	$X^2-14Y^2=57600(4^t)^2$	$(30x_{n+3}-26910X_{n+1},\ 7192x_{n+1}-8x_{n+3})$
3	$X^2-Y^2=4(4^t)^2$	$(112x_{n+1}-30y_{n+1},\ 30x_{n+1}-8Y_{n+1})$
4	$X^2-14X^2=900(4^t)^2$	$(30y_{n+2}-3360x_{n+1},\ 898x_{n+1}-8y_{n+2})$
5	$X^2-14Y^2=806404(4^t)^2$	$(30y_{n+3}-100688x_{n+1},\ 26910x_{n+1}-8y_{n+3})$
6	$X^2-14Y^2=64(4^t)^2$	$(898x_{n+3}-26910x_{n+2},\ 7192x_{n+2}-240x_{n+3})$
7	$X^2-14Y^2=900(4^t)^2$	$(898y_{n+1}-112x_{n+2},\ 30x_{n+2}-240y_{n+1})$
8	$X^2-14Y^2=4(4^t)^2$	$(898y_{n+2}-3360x_{n+2},\ 898x_{n+2}-240y_{n+2})$
9	$X^2-14Y^2=900(4^t)^2$	$(898y_{n+3}-100688x_{n+2},\ 26910x_{n+2}-240y_{n+3})$
10	$X^2-14Y^2=806404(4^t)^2$	$(26910y_{n+1}-112x_{n+3},\ 30x_{n+3}-7192y_{n+1})$
11	$X^2-14Y^2=900(4^t)^2$	$(26910y_{n+2}-3360x_{n+3},\ 898x_{n+3}-7192y_{n+2})$
12	$X^2-14Y^2=4(4^t)^2$	$(26910y_{n+3}-100688x_{n+3},\ 26910x_{n+3}-7192y_{n+3})$
13	$196X^2-14Y^2=12544(4^t)^2$	$(240y_{n+1}-8y_{n+2},\ 30y_{n+2}-898y_{n+1})$
14	$196X^2-14Y^2=5644800(4^t)^2$	$(7192y_{n+1}-8y_{n+3},\ 30y_{n+3}-26910y_{n+1})$
15	$3136X^2-224Y^2=200704(4^t)^2$	$(7192y_{n+2}-240y_{n+3},\ 898y_{n+3}-26910y_{n+2})$

2) Employing linear combination among the solutions of (1), one may generate integer solutions for other choices of parabolas which are presented in table 3 below

Table 3: Parabolas

S.NO	Parabola	(X,Y)
1	$4^t X+56Y^2=8(4^t)^2$	$(898x_{2n+2}-30x_{2n+3},\ 60x_{n+1}-2x_{n+2})$
2	$120(4^t)X-14Y^2=28800(4^t)^2$	$(30x_{2n+4}-26910x_{2n+2},\ 7192x_{n+1}-8x_{n+3})$
3	$(4^t)X+Y^2=2(4^t)^2$	$(112x_{2n+2}-30y_{2n+2},\ 30x_{n+1}-8y_{n+1})$
4	$449(4^t)X-14Y^2=403202(4^t)^2$	$(30y_{2n+4}-100688x_{2n+2},\ 26910x_{n+1}-8y_{n+3})$
5	$4(4^t)X-14Y^2=32(4^t)^2$	$(898x_{2n+4}-26910x_{2n+3},\ 7192x_{n+2}-240x_{n+3})$
6	$15(4^t)X-14Y^2=450(4^t)^2$	$(30y_{2n+3}-3360x_{2n+2},\ 898x_{n+1}-8y_{n+2})$
7	$15(4^t)X-14Y^2=450(4^t)^2$	$(898y_{2n+2}-112x_{2n+3},\ 30x_{n+2}-240y_{n+1})$
8	$(4^t)X-14Y^2=2(4^t)^2$	$(898y_{2n+3}-3360x_{2n+3},\ 898x_{n+2}-240y_{n+2})$
9	$15(4^t)X-14Y^2=450(4^t)^2$	$(898y_{2n+4}-100688x_{2n+3},\ 26910x_{n+2}-240y_{n+3})$
10	$449(4^t)X-14Y^2=403202(4^t)^2$	$(26910y_{2n+2}-112x_{2n+4},\ 30x_{n+3}-7192y_{n+1})$

11	$15(4^t)X-14Y^2=450(4^t)^2$	$(26910y_{2n+3}-3360_{2n+4}, 898x_{n+3}-7192y_{n+2})$
12	$(4^t)X-14Y^2=2(4^t)^2$	$(26910_{2n+4}-100688x_{2n+4}, 26910x_{n+3}-7192y_{n+3})$
13	$25320(4^t)X-14Y^2=5644800(4^t)^2$	$(7192y_{2n+2}-8y_{2n+4}, 30y_{n+3}-26910y_{n+1})$
14	$784(4^t)X-14Y^2=6272(4^t)^2$	$(240y_{2n+2}-8y_{2n+3}, 30y_{n+2}-898y_{n+1})$
15	$56(4^t)X-Y^2=448(4^t)^2$	$(7192y_{2n+3}-240y_{2n+4}, 898y_{n+3}-26910y_{n+2})$

3) Employing the following solutions (x,y), each of the following expressions among the special polygonal, pyramidal, star, pronic and centered polygonal number is congruent to under modulo 16

$$\left(\frac{3P^3{}_{y-2}}{t_{3,y-2}}\right)^2-\left(\frac{84P_x{}^3}{\Pr_{x+1}}\right)^2$$

$$\left(\frac{12P_y{}^5}{S_{y+1}-1}\right)^2-\left(\frac{504P_{X-2}{}^3}{S_{x-2}-1}\right)^2$$

$$\left(\frac{P_y{}^5}{t_{3,y}}\right)^2-\left(\frac{168P_x{}^5}{S_{x+1}-1}\right)^2$$

$$\left(\frac{2P_y{}^5}{Ct_{4,y}-1}\right)^2-\left(\frac{84P_x{}^5}{Ct_{6,x}}\right)^2$$

$$\left(\frac{4Pt_{n-3}}{P_{n-3}{}^3}\right)^2-\left(\frac{1848P_x{}^5}{Ct_{4,x}-1}\right)^2$$

IV. CONCLUSION

To conclude, one may search for other choices of positive Pell equations for finding their integer solutions with suitable properties.

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