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Optimization of Reactive Power Algorithm in Distribution Network Containing Wind Farm

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Abstract

A new strategy on reactive power optimization for power grid with wind farm integration is proposed. The mathematical model is built by using the optimal scenario analysis method and the modified particle swarm optimization algorithm (MPSO). The method about how to obtain the optimal scenarios is discussed. The optimal scenarios position for wind power is deduced based on the Wasserstein distance metric, and the occurrence probability is also studied simultaneously. In order to avoid falling into local optimum, the self-adapting mutagenic factor and mutation probability are designed in MPSO. Simulation examples show the effectiveness of MPSO. The mutagenic factor takes effect when the objective function value tends to be constant. Power loss and voltage stability margin are considered in the objective function. In initial phase of MPSO, the general particle swarm optimization algorithm (PSO) is processed, and this can guarantee the rapid convergence for the optimization procedure. After some iterations, the mutagenic factor begins to have an impact to ensure the global optimum can be obtained. The IEEE 69-bus distribution system is used to the experiment. In the experiment, the optimal scenarios position and scenario occurrence probability are worked out. Test results show that the new strategy is effective.

Keywords*: Reactive power optimization, Wind power, Optimal scenario, Particle swarm Optimization*

1. Introduction

Affected by the tight energy supply, the utilization of wind resources is being received attention across the globe. But the wind is a sort of random and intermittent energy [1], and when wind farm is injected into the distribution network, the load flow distribution of network is affected. The node voltage and the network loss are changed. This brings new problems in optimization of reactive power, and the existing optimization methods need to be modified.

In recent years, many scholars study the reactive power optimization in power system with wind power generators. In [2], Monte-Carlo simulation is used to process the reactive power optimization, but it cannot make response quickly and cannot change with power network's situation. In [3], the model and algorithm are put forward for the reactive power optimization of power grid embedded with multiple wind farms. The scenario analysis method is applied to describe the random outputs of wind power, and a scenario model is built for reactive power optimization. But, how to obtain the optimal scenarios position is not considered, and the method's effect is not good. In [4], a reactive power optimization algorithm is studied. The scenario analysis method and the quantum particle swarm optimization (QPSO) are proposed for reactive power optimization. The modified IEEE 33-bus system is used to demonstrate the effectiveness. But the scenario analysis process is too simple, and the complicated situations of wind power can not be fully considered.

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Of varied methods, the scenario analysis is an effective method to overcome the randomness of wind power, and it is often used to build the mathematical model of reactive power optimization. From the present study, we find that the scenarios are usually divided by experience in the scenario analysis method, and the study on how to obtain the optimal scenarios is seldom discussed. On the other hand, the optimization algorithm is very important in the reactive power optimization, and the particle swarm optimization (PSO) is often used to the optimization process. But PSO is easy to fall into local optimum, and the algorithm should be improved.

This paper puts forward a new algorithm for optimal reactive power. The algorithm is based on the optimal scenario analysis and the modified particle swarm optimization algorithm (MPSO). The formulas on how to get the optimal scenarios and the occurrence probability are deduced. In the MPOS, the self-adapting mutagenic factor and the mutation probability are designed and used, and the algorithm can avoid falling into the local optimum effectively.

2. The Optimal Scenarios Division

Scenario analysis technique is a method that handles the random and intermittence problem. It changes the uncertain factors to some certain scenarios in order to deal with the difficulty of processing. By scenario analysis, several certain scenarios can be obtained by decomposing the uncertain scenario, and all the scenarios are optimized respectively. By overlying each optimized result, the final optimize result can be acquired. This section firstly discusses the method on how to get optimal scenario, and then by the method, the optimal scenario formula for wind power is deduced.

2.1 The Method of Getting Optimal Scenario

The optimization method based on scenario analysis is to change a set of uncertain random variables to some certain scenarios. The uncertain random variables are expressed by a group of discrete probability expressions. The expression is (S_n, $\tilde{P}_n$) , $n=1,2,...,N$. S_n is the nth scenario. The $\tilde{P}_n$ is the probability of scenario occurrence. The n is the number of scenarios. In the process of acquiring the optimal scenario, the n must be determined at first, and then S_n and $\tilde{P}_n$ can be obtained when the optimization condition is met [5].

In this paper, the Wasserstein distance metric is used as the optimization condition. The S_n value and $\tilde{P}_n$ value are evaluated on the condition when the Wasserstein distance is minimum. The Wasserstein distance is given below:

$$d_w(P,\tilde{P}) = \sum_{n=1}^{N} \int_{(s_{n-1}+s_n)/2}^{(s_n+s_{n+1})/2} |x - s_n| \, dP(x) \tag{1}$$

Here $d_w(P, \tilde{P})$ is the Wasserstein distance; $P(\cdot)$ and $\tilde{P}(\cdot)$ are respectively the continuous and the discrete probability distribution function. The $S_1, S_2,... S_N$ are the scenarios for $P(x)$.

The optimal scenario can be acquired by the next two steps: first, the optimal scenario position S_n must be determined, and then $\tilde{P}_n$ can be obtained by the condition that Wasserstein distance is minimum. Based on the [5], the optimal scenario position S_n ($n=1,2, ..., N$) for the probability density function $f(x)$ can be evaluated by the next formula:

$$\int_{-\infty}^{s_n} \sqrt{f(x)} dx = \frac{2n-1}{2N} \int_{-\infty}^{+\infty} \sqrt{f(x)} dx \tag{2}$$

The probability of scenario occurrence $\tilde{P}_n$ can be obtained by next formulas:

$$p_n := \int_{(s_{n-1}+s_n)/2}^{(s_n+s_{n+1})/2} f(x)dx \qquad n=1,2,\ldots,N \tag{3}$$

Here $s_0 \to -\infty, s_{N+1} \to +\infty$.

Above formulas discuss the general method to get optimal scenario for a continuous discrete probability distribution function. To wind power, the optimal scenario can be deduced based on above formulas, and the concrete procession is illustrated as below.

2.2 The Optimal Scenario for Wind Power

The wind speed is related to the wind turbine generator's output power. The relationship between wind speed and output power is shown in Figure 1 [6].

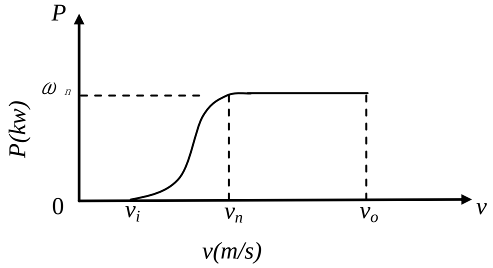

Figure 1. Power Output Curve of Wind Turbine

The cut-in wind speed is v_i , and the rated wind speed is v_n ; v_o represents the cut-out wind speed. The rated output of wind turbine is ω_n [7, 8] .

When the wind power $\omega \in (0, \omega_n)$, the probability density function for wind power can be given:

$$f(\omega) = f(v_i < V < v_n) = c_2 k t^{k-1} e^{-t^k} \tag{4}$$

Where $t=(t+h/\omega_n)$; $(v_i/c)= c_2\omega + c_1$; $c_1=v_i/c$; $c_2=(v_i/c)(h/\omega_n)$. The c is the scale parameter, $c>0$; k represents the shape parameter, $k>0$.

Above formula is substituted into the right of formula (2), and the next formula is obtained:

$$(\frac{2n-1}{2N})\int_0^{+\infty}\sqrt{f(\omega)}d\omega = (\frac{2n-1}{2N})\frac{(c_2k)^{1/2}}{c_2}\int_{c_1}^{c_1+c_2} t^{\frac{k-1}{2}} e^{-\frac{t^k}{2}} dt \tag{5}$$

Let $a=(k+1)/(2k)$, $x=t^k/2$, the next formula is obtained:

$$(\frac{2n-1}{2N})\int_0^{+\infty}\sqrt{f(\omega)}d\omega = (\frac{2n-1}{2N}) * 2^a * (c_2k)^{1/2} * \int_{c_1}^{c_1+c_2} x^{a-1}e^{-x}dx \tag{6}$$

The $c_3=2^a(kc_2)^{-1/2}$. The incomplete gamma function is $\Gamma(\mathrm{x},\mathrm{a})\Gamma(\mathrm{a}) = \int_0^a x^{a-1}e^{-x}dx$, the next formula is obtained:

$$(\frac{2n-1}{2N})\int_0^{+\infty}\sqrt{f(\omega)}d\omega = (\frac{2n-1}{2N})c_3\Gamma(\mathrm{a})[\Gamma(\mathrm{c_1}+\mathrm{c_2},\mathrm{a}) - \Gamma(\mathrm{c_1},\mathrm{a})] \tag{7}$$

In the same way, the left of formula (2) can be shown:

$$\int_0^{s_n}\sqrt{f(\omega)}d\omega = c_3\Gamma(\mathrm{a})[\Gamma(\mathrm{c_1}+\mathrm{c_2}s_n,\mathrm{a}) - \Gamma(\mathrm{c_1},\mathrm{a})] \tag{8}$$

So, formula (2) can be expressed:

$$c_3\Gamma(\mathrm{a})[\Gamma(\mathrm{c}_1+\mathrm{c}_2 s_\mathrm{n},\mathrm{a})-\Gamma(\mathrm{c}_1,\mathrm{a})]=(\frac{2n-1}{2N})c_3\Gamma(\mathrm{a})[\Gamma(\mathrm{c}_1+\mathrm{c}_2,\mathrm{a})-\Gamma(\mathrm{c}_1,\mathrm{a})] \tag{9}$$

Through above analysis, we can see that the optimal scenario position S_n can be evaluated by formula (9), and the scenario occurrence probability $\tilde{P}_n$ can be evaluated by formula (3).

By Figure 1, we can obtain three kinds of scenarios. The first is the scenario of rated output ($v_n<v<v_o$), the second is the scenario of owe rated output ($v_i<v<v_n$), and the third is the scenario of zero output ($v<v_i$ or $v>v_o$). Ulteriorly, the scenario of owe rated output is divided into four scenarios in this paper, and each optimal scenario position (S_n) can be determined by formula (9); the scenario occurrence probability ($\tilde{P}_n$) can be calculated by formula (3).

3. The Modified Particle Swarm Optimization Algorithm (MPSO)

3.1. The Introduce of MPSO

The particle swarm optimization algorithm (PSO) has been widely used [9][10]. Supposing the number of the particle population is u, and u is the set of k-dimensional vectors. The ith vector is $x_i=(x_{i1}, x_{i2}, \ldots, x_{ik})$. The optimal solution of each particle is $p_{besti}=(p_{i1},p_{i2}, \ldots ,p_k)$; the optimal solution for particle population is $p_{gbest}=(p_{gbest1}, p_{gbest2}, \ldots, p_{gbestk})$; the speed of ith particle is $v_i=(v_{i1}, v_{i2},\ldots, v_{ik})$. The speed and the position for each particle are updated by the following formula:

$$v_{id}^{n+1}=\delta v_{id}^{n}+c_1 r_1(p_{bestid}^{n}-x_{id}^{n})+c_2 r_2(p_{gbestd}-x_{id}^{n}) \tag{10}$$

$$x_{id}^{n+1}=x_{id}^{n}+v_{id}^{n+1} \tag{11}$$

Formula (10) is the update of speed, and formula (11) is the update of position. The d is the number of dimension; δ is inertia weight; n is iterations; r_1 and r_2 are random number between 0 and 1; c_1 and c_2 are acceleration coefficient. In each iteration, v^{n}_{id} is updated by v^{n+1}_{id}, and the x^{n}_{id} is updated toward x^{n+1}_{id} at speed of v^{n+1}_{id}.

PSO algorithm has fast convergence speed, but the local optimal problem limits its application. Adding the mutagenic factor to the PSO can increase the dispensability of the particle swarm, and the premature convergence can be restrained. The MPSO is put forward based on PSO, and the mutagenic factor is designed to avoid falling into local optimum. When the convergence result tends to be constant, the mutagenic factor is added to formula (11), and formula (11) is replaced by next formulas:

$$x_{id}^{n+1}=x_{id}^{n}+v_{id}^{n+1}+r\cdot\frac{10}{n}\cdot\sigma \tag{12}$$

$$\sigma=(\sum_{d=1}^{k}x_{id}{}^{n})/k \tag{13}$$

The $r(10/n)\sigma$ is the mutagenic factor. The r is a random number between 0 and 1; n is iterations. By formula (13), we know that σ is the average of the x^{n}_{id}, $d=(1,2,\ldots, k)$.

From above formulas, it can be seen that the mutagenic factor is smaller with the increasing of iterations. So, the effect of the mutagenic factor is that x^{n}_{id} has a bigger variation in the initial phase, and with the iterations increasing, the degree of variation is smaller. That is conducive to obtain a convergence result.

The mutation probability P_m is designed as below:

$$P_m=0.1+0.02(n_{\max}/n) \tag{14}$$

Here n is iterations, and n_{max} is the maximum iterations.

From formula (14) we can see that P_m is smaller with the increased of n. When the n is equal to n_{max}, P_m reaches the minimum.

The strategy of MPSO is that in the beginning stages of the algorithm, the process is carried out according to formula (10)(11) in order to obtain a fast convergence speed. When the result tends to be constant, the above mutagenic factor should be added according to formula (12), and the above P_m is also needed. By using mutagenic factor, diversity of particles is ensured, and premature convergence is overcome availably.

Figure 2 shows the working process for MPSO. The n_m is the iteration that mutagenic factor should be added.

3.2. The Simulation Examples for MPSO

Two commonly used functions, the Levy function and the Pathological function, are used in the simulation experiment in order to prove the validity of MPSO.

$$y = \sin^2(3\pi x_i) + \sum_{i=1}^{n-1}(x_i - 1)^2(1+\sin^2(3\pi x_{i+1})) + (x_n - 1)\,(1+\sin^2(2\pi x_n)) \tag{15}$$

$$y = \sum_{i=1}^{n-1}(0.5+(\sin\sqrt{100x_i^{\,2} + x^2{}_{i+1}} - 0.5)/(1+0.001(x^2{}_i - 2x_i x_{i+1} + x_{i+1})^2)) \tag{16}$$

Formula (15) is the Levy function, and formula (16) is the Pathological function.

The operating parameters for MPSO are: δ =0.7; *c1*=*c2*=1.5; population quantity is 100; the dimension of each particle is 50; the total iterations are 800. For the Levy function, the mutagenic factor is added at 300 iteration, n_m=300. For the Pathological function, the mutagenic factor is added at the beginning of MPSO, $n_m=0$. To above two formulas, Figure 3 and Figure 4 give the contrast results of PSO and MPSO. From the contrast results we can see that the global convergence results are obtained by MPSO algorithm and the MPSO is more effective.

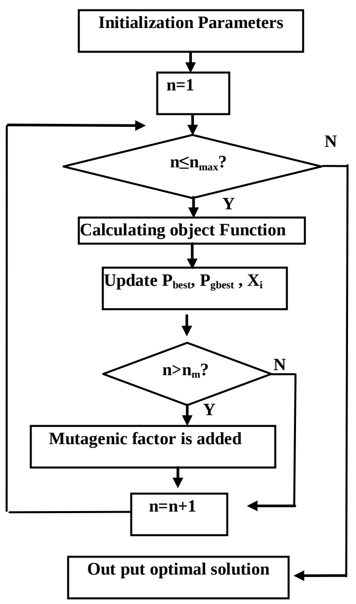

Figure 2. Flow of MPSO

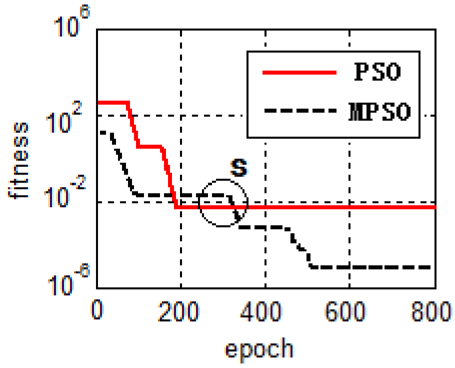

Figure 3. Result of Levy Function

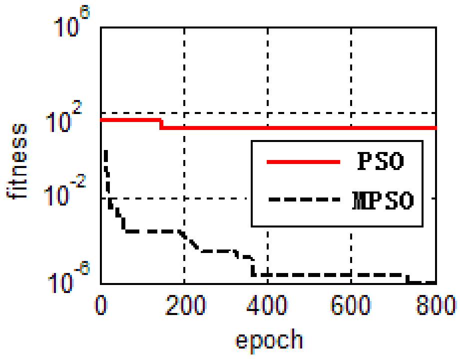

Figure 4. Result of Pathological Function

4. The Mathematical Model for Reactive Power Optimization

4.1 The Objective Function

The multi-objective optimization function is set up in this paper. Power losses (P_{loss}) and stability margin of static voltage (η) are considered. The ultimate aim is obtaining the minimum power losses and the stability margin. Relevant parameters are redefined base on the scenario occurrence probability.

$$\mu_1 = \sum_{k=1}^{n} p_k * p_{loss}^k \tag{15}$$

$$\mu_2 = \sum_{k=1}^{n} p_k * \eta_k \tag{16}$$

The n is the total number of scenarios, and $P^k{}_{loss}$ is the power losse for the kth scenario; η_k is the stability margin of static voltage for the kth scenario, and p_k is the occurrence probability of the kth scenario. By the normalization, above two formulas are given below:

$$P_{loss}^* = (\mu_1 - \mu_{1\min})/(\mu_{1\max} - \mu_{1\min}) \tag{17}$$

$$\eta^* = (\mu_{2\max} - \mu_2)/(\mu_{2\max} - \mu_{2\min}) \tag{18}$$

The next formula is used as the final objective function:

$$\min F = t_1 * P_{loss}^* + t_2 * \eta^* \tag{19}$$

The t_1 and t_2 are weighing factors.

4.2 The Constraints

The constraints for active power and reactive power of each node are:

$$P_i = U_i \sum_{j=1}^{n} U_j (G_{ij} \cos\delta_{ij} + B_{ij} \sin\delta_{ij}) \tag{20}$$

$$Q_i = U_i \sum_{j=1}^{n} U_j (G_{ij} \sin\delta_{ij} - B_{ij} \cos\delta_{ij}) \tag{21}$$

The P_i and Q_i are the active power and reactive power of node i; U_i *and* U_i are the voltage of node i and j, and δ_{ij} is the angular phase difference of node i and node j. G_{ij} *and* B_{ij} are the conductance and susceptance for node i and node j.
In wind farm, the values of P_i and Q_i are respectively depend on wind speed and generator voltage [11].
The control variable constraints are given below:

$$Q_{G\min} \le Q_G \le Q_{G\max} \tag{22}$$

$$T_{\min} \le T \le T_{\max} \tag{23}$$

$$U_{G\min} \le U_G \le U_{G\max} \tag{24}$$

$$U_{i\min} \le U_i \le U_{i\max} \tag{25}$$

$$C_{\min} \le C \le C_{\max} \tag{26}$$

The Q_g is the reactive power of generator, and U_G is the generator terminal voltage. T is the ratio of variable-voltage transformer, and C is the susceptance value. U_i is the node voltage.

4.3 Coding Format

The capacitor set numbers (T) and adjustable transformer tap values(C) are used to coded, and integer coding is adopted. The coding format is shown as below:

$$x = [T_1 T_2 \dots T_i \mid C_1 C_2 \dots C_j] \tag{27}$$

The x is one particle. The T is the adjustable transformer tap, and C represents value and capacitor set number. The i and j respectively represent the total numbers of the adjustable transformer and the total numbers of the capacitance bank.

5. Experiment and Analysis

The IEEE 69-bus distribution system, which is shown in Figure 5, is adopted to verify the effectiveness of MPSO. The detailed parameters data for IEEE 69-bus are given in [4]. The five nodes (11, 27, 35, 47, 54 and 69) are used to cut and throw capacitor banks, and the capacitor bank of every capacity is 50kvar. Ten every capacitor banks compose a group. The size of particle swarm is 40. The maximum iteration (n_{max}) is 120; $c_1=c_2=2$.

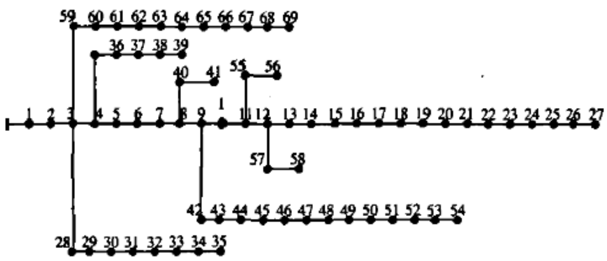

Figure 5. IEEE 69-Bus Distribution System

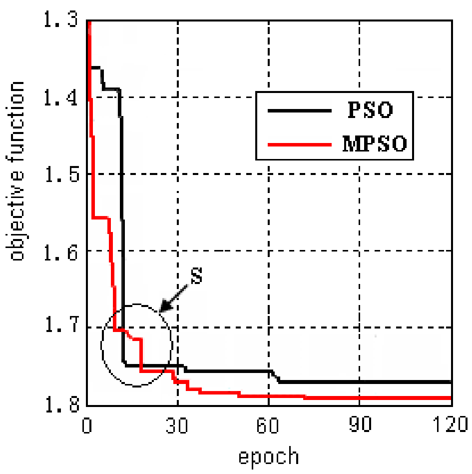

Figure 6. Comparison of two Algorithms

At the node of 54, a wind generator (rated voltage is 690v) is injected. The capacity of the wind generator rated is 600kw. The cut-in wind speed is 5m/s. The cut-out wind speed is 45m/s. The rated wind speed is 15m/s.

When the mutagenic factor takes effect is a problem that deserves to be discussed. By observing Figure 3 and Figure 6 we found that there usually is the area S before the algorithm trapping in local optimum. If the mutagenic factor is added at the iteration in the area S, the local optimum result can be avoided. By studying the experimental curves, the area S is easily found. By the optimal curve of this paper, we find the area S is near the 20 iterations. So, $n_m=20$.

According to section 2.2, the owe rated output scenario is consisted of four scenarios. The four scenarios is expressed by S_n $(n=2,\ldots,5)$, and the occurrence probability is P_n $(n=2,\ldots,5)$. The zero output scenario and occurrence probability are S_1 and P_1. The rated output scenario and it's occurrence probability are S_6 and P_6. By the formula (9) and formula (3), we can obtain the results that are shown in Table 1.

Table 1. The Scenarios and their Occurrence Probability

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
S_n	0.0000	0.0768	0.2701	0.5000	0.8021	1.0000
P_n	0.1049	0.0792	0.1054	0.1601	0.1759	0.3745

Figure 6 shows the comparison curves for MPSO and PSO. From curves in Figure 5 we can see that both the two algorithms can converge rapidly in the initial phases. But, the PSO has trapped into local optima after 20 epochs, and the MPSO can obtain the result of global convergence.

Table 2 gives the results of reactive power optimization. In optimization schemes, the number that is shown in the brackets is capacitor set numbers, and the number outside the brackets is node numbering. The final optimum result is represented by "opt". In the final optimum results, the occurrence probability for each scenario is considered.

Table 2. Optimization results

	Optimization schemes						**power loss(kw)**	**stability margin of static voltage**
scenario **1**	11(7)	27(7)	35(2)	47(6)	54(9)	69(8)	83.52	0.01281
scenario **2**	11(8)	27(7)	35(3)	47(6)	54(8)	69(8)	96.02	0.01281
scenario **3**	11(8)	27(8)	35(4)	47(7)	54(9)	69(6)	115.01	0.01289
scenario **4**	11(9)	27(10)	35(5)	47(8)	54(10)	69(7)	124.33	0.01293
scenario **5**	11(9)	27(9)	35(4)	47(8)	54(10)	69(5)	114.99	0.01288
scenario **6**	11(10)	27(9)	35(4)	47(6)	54(9)	69(6)	107.13	0.01295
opt	11(10)	27(9)	35(3)	47(6)	54(10)	69(6)	106.51	0.01283

In Table 2, the stability margin of static voltage and the power loss for opt and all scenarios are given. As can be seen from table 2, though the results of opt are not the optimum value compared with the other scenarios, but comprehensive evaluation results of opt are optimal.

6. Conclusions

In this paper, the reactive power optimization for power network with wind farm is proposed. The mathematical model based on optimal scenario classification and the modified particle swarm optimization is presented. To solve the intermittent and uncontrollability that brought by wind farm injection, the method of optimal scenario analysis is used. The optimal scenarios position for wind power is deduced based on the Wasserstein distance metric, and the occurrence probability of scenario formula is given. The PSO is improved to avoid involving local optimum. The self-adapting mutagenic factor and P_m are designed in MPSO. When the algorithm tends to local optimum, the mutagenic factor begins to take effect, and the dispersion of particle swarm can be ensured. Experimental results have shown that MPSO not only can solve the problem brought by the wind farm integration, but also has better global astringency.

Acknowledgements

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