## Subspectral Editing with a Multiple Quantum Trap of $IS_n$ Spin Systems by Using Product Operator Theory

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Product operator theory was often used to describe analytically multipulse NMR experiments for weakly coupled spin systems. In this study first we introduce the descriptions of subspectral editing with a multiple quantum trap NMR spectra for  $IS_n$  (I = 1/2, S = 5/2 with n = 1, 2, 3) spin systems by using product operator formalism. These theoretical investigations lead us to form the general expressions for the intensities of the spin -1/2nuclei coupled to the nuclei with spin  $\geq 5/2$ . The obtained results can be used for the spectral editing in both liquid-state and solid-state NMR experiments. Furthermore, in order to satisfy the obtained analytical expressions for signal intensities we add the presentation of analytically description of subspectral editing with a multiple quantum trap sequence for weakly coupled IS (I = 1/2, S = 7/2) spin system.

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#### 1. Introduction

Subspectral editing using a multiple quantum trap (namely SEMUT sequence) has been proposed as an alternative method for subspectral editing of <sup>13</sup>C NMR spectra [1]. This pulse sequence has mainly two advantages. Firstly, it contains fewer pulses than polarization technique distortionless enhancement po-

(73)

larization transfer (DEPT) and SEMUT sequence includes quaternaries for the determination of proton multiplicities in <sup>13</sup>C NMR while DEPT can produce only CH, CH<sub>2</sub>, and CH<sub>3</sub> subspectra [2]. Secondly, it is most convenient experiment to be analyzed by using product operator theory as a simple quantum mechanical method [3, 4]. In this framework recently, SEMUT sequence has been described analytically for weakly coupled  $IS_n$  (I = 1/2, S = 1 and 3/2 with n = 1, 2, 3) spin systems by using product operator formalism [5, 6].

On the other hand, it is a well known fact that approximately 74% of NMR active nuclei in the periodic table have a spin greater than 1/2. For this reason a somewhat unusual two-spin system involving a spin S = 5/2 (or  $\geq 5/2$ ) could be interesting for some spectral editing experiments in particular when one considers the solid-state analogue of the SEMUT experiment [7].

In the present work first we introduce the analytical descriptions of SEMUT sequence for weakly coupled  $IS_n$  (I = 1/2, S = 5/2 with n = 1, 2, 3) spin systems by using product operator theory. Furthermore, we resume the similar results for weakly coupled  $IS_n$   $(I = 1/2, S \ge 1/2; n = 1, 2, 3)$  spin systems in Table which includes the earlier obtained results for  $IS_n$  (I = 1/2, S = 1/2 and 3/2; n = 1, 2, 3)spin systems in the mentioned pulse sequence. Later we present the description of SEMUT sequence for another weakly coupled spin system IS (I = 1/2, S = 7/2)in Appendix for the purpose of confirming the signal intensity in formed Table.

## 2. The evolutions of product operators under spin-spin coupling Hamiltonian for $IS_n$ (I = 1/2, S = 5/2) spin system and application to SEMUT sequence

For the analysis of multipulse experiments by using product operator formalism when a spin I = 1/2 is coupled to a spin S = 5/2, under scalar coupling it is convenient to consider the decomposition of I = 1/2 spin multiplicity into in-phase and anti-phase coherence with the inner and outer transitions of multiplet [4, 8–10]. This leads us to consider the operators  $I_x$ ,  $I_y$ ,  $I_xS_z$  and  $I_yS_z$  as some of the product operators for IS (I = 1/2, S = 5/2) spin system. By considering the Hausdorff formula for the evolutions of the mentioned product operators under spin-spin coupling Hamiltonian a shorthand notation can be obtained as follows [9]:

$$I_{x} \xrightarrow{2\pi J I_{z} S_{z} t} I_{x} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) + \frac{2}{5} I_{y} S_{z} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t)$$
  
+ $I_{x} E_{s}(\pm \frac{3}{2}) \cos(3\pi J t) + \frac{2}{3} I_{y} S_{z} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t)$   
+ $I_{x} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) + 2 I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \sin(\pi J t),$  (1a)

$$I_{y} \stackrel{2\pi J I_{z} S_{z} t}{\longrightarrow} I_{y} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) - \frac{2}{5} I_{x} S_{z} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) + I_{y} E_{s}(\pm \frac{3}{2}) \cos(3\pi J t) - \frac{2}{3} I_{x} S_{z} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) + I_{y} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) - 2I_{x} S_{z} E_{s}(\pm \frac{1}{2}) \sin(\pi J t),$$
(1b)  
$$I_{x} S_{z} \stackrel{2\pi J I_{z} S_{z} t}{\longrightarrow} I_{x} S_{z} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) + \frac{5}{2} I_{y} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) + I_{x} S_{z} E_{s}(\pm \frac{3}{2}) \cos(3\pi J t) + \frac{3}{2} I_{y} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) + I_{x} S_{z} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) + \frac{1}{2} I_{y} E_{s}(\pm \frac{1}{2}) \sin(\pi J t),$$
(1c)  
$$I_{y} S_{z} \stackrel{2\pi J I_{z} S_{z} t}{\longrightarrow} I_{y} S_{z} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) - \frac{5}{2} I_{x} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) + I_{y} S_{z} E_{s}(\pm \frac{3}{2}) \cos(3\pi J t) - \frac{3}{2} I_{x} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) - \frac{3}{2} I_{x} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) - \frac{1}{2} I_{x} E_{s}(\pm \frac{1}{2}) \sin(\pi J t),$$
(1d)

We used these expressions for the analytical description of SEMUT sequence within the framework of product operator formalism. SEMUT sequence is shown in Fig. 1. The numbers labelled in Fig. 1 indicate all single stages of the density matrix operators in SEMUT pulse sequences.



Fig. 1. SEMUT pulse sequence  $(\tau = 1/(2J))$ .

For IS (I = 1/2, S = 5/2) weakly coupled spin system the density matrix operators are as follows: in equilibrium state we have  $\sigma_0 = I_z$  and after the first pulse  $\sigma_1 = -I_y$ . During  $\tau$  interval, the density matrix operator is

$$\sigma_2 = -I_y E_s(\pm \frac{5}{2})C_{5J} + \frac{2}{5}I_x S_z E_s(\pm \frac{5}{2})S_{5J}$$
$$-I_y E_s(\pm \frac{3}{2})C_{3J} + \frac{2}{5}I_x S_z E_s(\pm \frac{3}{2})S_{3J}$$
$$-I_y E_s(\pm \frac{1}{2})C_J + \frac{2}{5}I_x S_z E_s(\pm \frac{1}{2})S_J,$$

where  $C_{nJ} = \cos(n\pi J\tau)$  and  $S_{nJ} = \sin(n\pi J\tau)$ . For  $\tau = 1/(2J)$  we take the values  $C_J = C_{3J} = C_{5J} = 0$  and  $S_J = S_{5J} = 1$ ,  $S_{3J} = -1$  and thus  $\sigma_2$  becomes

$$\sigma_2 = \frac{2}{5} I_x S_z E_s(\pm \frac{5}{2}) - \frac{2}{3} I_x S_z E_s(\pm \frac{3}{2}) + 2 I_x S_z E_s(\pm \frac{1}{2}).$$
(2)

Then, after the applications of  $(180^{\circ})_x$  and  $(\theta)_x$  pulses we obtain

$$\sigma_3 = \frac{2}{5} I_x S_z E_s(\pm \frac{5}{2}) C_\theta - \frac{2}{3} I_x S_z E_s(\pm \frac{3}{2}) C_\theta + 2 I_x S_z E_s(\pm \frac{1}{2}) C_\theta, \qquad (3)$$

where  $C_{\theta} = \cos \theta$ . During  $\tau$  evolution time, we get

$$\sigma \xrightarrow{2\pi J I_z S_z} \sigma_4,$$
  

$$\sigma_4 = \frac{2}{5} I_x S_z E_s(\pm \frac{5}{2}) C_\theta C_{5J} + I_y E_s(\pm \frac{5}{2}) C_\theta S_{5J}$$
  

$$-\frac{2}{3} I_x S_z E_s(\pm \frac{3}{2}) C_\theta C_{3J} - I_y E_s(\pm \frac{3}{2}) C_\theta S_{3J}$$
  

$$+2 I_x S_z E_s(\pm \frac{1}{2}) C_\theta C_J + I_y E_s(\pm \frac{1}{2}) C_\theta S_J.$$
(4)

By taking the values  $C_J = C_{3J} = C_{5J} = 0$ ,  $S_J = S_{5J} = 1$  and  $S_{3J} = -1$  for  $\tau = 1/(2J)$  we get

$$\sigma_4 = I_y E_s(\pm \frac{5}{2}) C_\theta + I_y E_s(\pm \frac{3}{2}) C_\theta + I_y E_s(\pm \frac{1}{2}) C_\theta.$$
(5)

During  $\tau$  between stages 3 and 4 in Fig. 1, relaxation and effect of chemical shift Hamiltonian on the evolutions of product operators can be disregarded. But during detection time, t, the chemical shift effect exists. As a matter of fact, the calculation can be stopped at point four because of the density operator at this point. On the other hand, the signal is detected from y-axis and since the contributions to the observable signals becomes only including  $I_y$  product operator terms, the magnetization is proportional to  $\langle I_y \rangle$ , that is,

$$M_y(t) \sim \langle I_y \rangle = \text{Tr}[I_y \sigma_4]. \tag{6}$$

For IS (I = 1/2, S = 5/2) spin system by substituting Eq. (7) into Eq. (8) we have the coefficients

$$\operatorname{Tr}[I_y I_y E_s(\pm \frac{5}{2})] = \operatorname{Tr}[I_y I_y E_s(\pm \frac{3}{2})] = \operatorname{Tr}[I_y I_y E_s(\pm \frac{1}{2})] = 1.$$
(7)  
we obtain

Thus we obtain

$$\langle I_y \rangle (IS) = 3C_\theta. \tag{8}$$

For  $IS_2$  (I = 1/2, S = 5/2) spin systems by following the same calculations steps we obtain the observable signal as

$$\langle I_y \rangle (IS_2) = 18C_\theta^2. \tag{9}$$

In a similar way, for  $IS_3$  (I = 1/2, S = 5/2) spin system the observable signal becomes

$$\langle I_y \rangle (IS_3) = 4 \times 27 C_\theta^3. \tag{10}$$

#### 3. Discussion and conclusions

Considering Eqs. (8)–(10) we can study the dependencies of observable signal intensities on the pulse angle  $\theta$  (Fig. 2). In Fig. 2 the unnormalized values are used and if we denote the  $IS_n$  (I = 1/2, S = 5/2) spin systems as  $XY_n$  (for instance,  $X = {}^{13}$ C), the relative intensities of  ${}^{13}$ C SEMUT NMR spectra can be observed separately for every single group. In the case of  $\theta = 90^{\circ}$  or 270° only quaternary carbons are observed. From Fig. 2 it is easily seen that the relative intensities for CY, CY<sub>2</sub>, and CY<sub>3</sub> groups are the same at the angle 180°.



Fig. 2. The plot of the signal intensities as a function of the pulse angle  $\theta$ .

On the other hand, the obtained intensity values exhibit a significant proportionality to the results of weakly coupled IS (I = 1/2, S = 1/2 and 3/2; n = 1, 2, 3) spin systems by using product operator formalism in the subspectral editing <sup>13</sup>C NMR SEMUT spectra [1, 6]. Based on this proportionality, the intensities of the observable signals for weakly coupled half-integer spin systems are listed in Table.

From Table, we can derive an expression between the total signal intensities and the dimensions in the matrix representations of S spin operators as

$$I = n \left(\frac{N}{2}\right)^n \cos^n \theta \qquad \text{for } n = 1,2 \tag{11}$$

and

$$I = (n+1)\left(\frac{N}{2}\right)^n \cos^n \theta \qquad \text{for } n = 3,$$
(12)

where N is the dimension of the matrix representation of S spin operator.

#### TABLE

The obtained signal intensities in the analytical descriptions of SEMUT sequence by using product operator theory for weakly coupled  $IS_n$  (I = 1/2, S = 1/2, 3/2, 5/2, 7/2, and 9/2; n = 1, 2, 3) spin systems.

Spin	Coefficients	$S = 1/2^{a}$	$S = 3/2^{b}$	S = 5/2	S = 7/2	S = 9/2
system		, ,	,	,	,	,
IS	$\cos \theta$	1	2(=1.2)	3(=1.3)	4(=1.4)	5(=1.5)
$IS_2$	$\cos^2 \theta$	2	$8(=2.2^2)$	$18(=2.3^2)$	$32(=2.4^2)$	$50(=2.5^2)$
$IS_3$	$\cos^3  heta$	4	$32(=4.2^3)$	$108(=4.3^3)$	$256(=4.4^3)$	$500(=4.5^3)$

<sup>a</sup>Taken from Ref. [1] and <sup>b</sup>taken from Ref. [6].

As the conclusion we can express that although the spin systems involving the spin  $S \geq 5/2$  are rather unusual for the spectral editing experiments the product operator formalism became a crucial method to describe analytically multidimensional and multipulse sequences for scalar coupled spin systems in both solvent and dilute-solids NMR.

#### Appendix

# The analytical description of SEMUT sequence for weakly coupled IS (I = 1/2, S = 7/2) spin system by using product operator theory

According to the decomposition mentioned in Sec. 1, the unitary matrix representation of S = 7/2 spin operator can be written as

$$E_s = E_s(\pm \frac{7}{2}) + E_s(\pm \frac{5}{2}) + E_s(\pm \frac{3}{2}) + E_s(\pm \frac{1}{2}), \tag{A.1}$$

where

Thus the product operator  $I_x$  can be defined as

$$I_x = I_x \otimes E_s$$
$$= I_x \otimes E_s(\pm \frac{7}{2}) + I_x \otimes E_s(\pm \frac{5}{2}) + I_x \otimes E_s(\pm \frac{3}{2}) + I_x \otimes E_s(\pm \frac{1}{2}).$$
(A.3)

In order to express the evolutions of operator  $I_x$  under spin-spin coupling Hamiltonian,  $H_J = 2\pi J I_z S_z$ , we should use the Hausdorff formula and the conditions

$$S_{z}^{n} E_{s}(\pm \frac{\tau}{2}) = \frac{49}{4} S_{z}^{n-2} E_{s}(\pm \frac{\tau}{2}), \qquad n \ge 2,$$

$$S_{z}^{n} E_{s}(\pm \frac{5}{2}) = \frac{25}{4} S_{z}^{n-2} E_{s}(\pm \frac{5}{2}), \qquad n \ge 2,$$

$$S_{z}^{n} E_{s}(\pm \frac{3}{2}) = \frac{9}{4} S_{z}^{n-2} E_{s}(\pm \frac{3}{2}), \qquad n \ge 2,$$

$$S_{z}^{n} E_{s}(\pm \frac{1}{2}) = \frac{1}{4} S_{z}^{n-2} E_{s}(\pm \frac{1}{2}), \qquad n \ge 2,$$
(A.4)

and we have

and

$$I_{x} \overset{2\pi JI_{z}S_{z}t}{\longrightarrow} I_{x}E_{s}(\pm \frac{7}{2})\cos(7\pi Jt) + \frac{2}{7}I_{y}S_{z}E_{s}(\pm \frac{7}{2})\sin(7\pi Jt) \\ + I_{x}E_{s}(\pm \frac{5}{2})\cos(5\pi Jt) + \frac{2}{5}I_{y}S_{z}E_{s}(\pm \frac{5}{2})\sin(5\pi Jt) \\ + I_{x}E_{s}(\pm \frac{3}{2})\cos(3\pi Jt) + \frac{2}{3}I_{y}S_{z}E_{s}(\pm \frac{3}{2})\sin(3\pi Jt) \\ + I_{x}E_{s}(\pm \frac{1}{2})\cos(\pi Jt) + 2I_{y}S_{z}E_{s}(\pm \frac{1}{2})\sin(\pi Jt),$$
(A.5a)

$$\begin{split} I_{y} & {}^{2\pi J I_{z} S_{z} t} I_{y} E_{s}(\pm \frac{\tau}{2}) \cos(7\pi J t) - \frac{2}{7} I_{x} S_{z} E_{s}(\pm \frac{\tau}{2}) \sin(7\pi J t) \\ & + I_{y} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) - \frac{2}{5} I_{x} S_{z} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) \\ & + I_{y} E_{s}(\pm \frac{3}{2}) \cos(3\pi J t) - \frac{2}{3} I_{x} S_{z} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) \\ & + I_{y} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) - 2 I_{x} S_{z} E_{s}(\pm \frac{1}{2}) \sin(\pi J t), \quad (A.5b) \\ I_{x} S_{z} & {}^{2\pi J I_{z} S_{z} t} I_{x} S_{z} E_{s}(\pm \frac{\tau}{2}) \cos(7\pi J t) + \frac{\tau}{2} I_{y} E_{s}(\pm \frac{\tau}{2}) \sin(7\pi J t) \\ & + I_{x} S_{z} E_{s}(\pm \frac{5}{2}) \cos(5\pi J t) + \frac{5}{2} I_{y} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) \\ & + I_{x} S_{z} E_{s}(\pm \frac{5}{2}) \cos(3\pi J t) + \frac{3}{2} I_{y} E_{s}(\pm \frac{3}{2}) \sin(3\pi J t) \\ & + I_{x} S_{z} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) + \frac{1}{2} I_{y} E_{s}(\pm \frac{1}{2}) \sin(\pi J t), \quad (A.5c) \\ I_{y} S_{z} & {}^{2\pi J I_{z} S_{z} t} I_{y} S_{z} E_{s}(\pm \frac{\tau}{2}) \cos(7\pi J t) - \frac{\tau}{2} I_{x} E_{s}(\pm \frac{\tau}{2}) \sin(7\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(3\pi J t) - \frac{5}{2} I_{x} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(3\pi J t) - \frac{5}{2} I_{x} E_{s}(\pm \frac{5}{2}) \sin(5\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(3\pi J t) - \frac{3}{2} I_{x} E_{s}(\pm \frac{1}{2}) \sin(3\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(3\pi J t) - \frac{3}{2} I_{x} E_{s}(\pm \frac{1}{2}) \sin(3\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(3\pi J t) - \frac{1}{2} I_{x} E_{s}(\pm \frac{1}{2}) \sin(3\pi J t) \\ & + I_{y} S_{z} E_{s}(\pm \frac{1}{2}) \cos(\pi J t) - \frac{1}{2} I_{x} E_{s}(\pm \frac{1}{2}) \sin(\pi J t). \end{aligned}$$

By following the same procedure within the text we obtain the observable signal as

$$\langle I_y \rangle (IS) = 4C_\theta. \tag{A.6}$$

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